Integrating Certified Lengths to Strengthen Metrology Network Uncertainty

Scott Sandwith
Dr. Joe Calkins
Introduction

- **Process**
  - Scaling 3D Metrology to reference temperature

- **Problem**
  - Coefficient of Thermal Expansion Compensation Uncertainty

- **Understanding**
  - Scale Bar/Length Traceability & Uncertainty

- **Solution**
  - Integrating Traceable Scale Lengths into USMN

- **Results**
  - Improved CTE Compensation with Uncertainty Analysis

- **Summary**
Process: Scaling 3D Metrology to Ref Temperature

Why Scale 3D Measurements?
- Object dimension is dependent on temperature
- Reference Temperature is 20°C (68°F)
- Nominals are given at reference temperature
- Objects are measured at temperatures other than at reference

Scale object measurements from actual to reference temperature
- Scale is dependent on:
  - Material Properties (CTE)
  - Temperature difference from reference
  - Object constraints
Thermal Length Compensation

Must scale measurements to reference temperature for comparison against nominals or between surveys

\[ L_i = L_0 (1 - \alpha \Delta T) \]

where:

- \( L_i \) = actual length at temperature
- \( L_0 \) = calibrated length at reference temperature
- \( \alpha \) = CTE for scale bar material (ppm/°C)
- \( \Delta T \) = temperature delta between actual and reference temperature (°C)

Objects change length as temperatures change.
Process: 3D Measurement Traceability

- **Survey scale is set with calibrated Temperature and CTE**
  - Thermocouples to measure object temperature ... e.g., ± 0.5°C (k = 2)
  - Published Material Type CTE e.g., ± 3-5% (k = 2)

- **Survey scale is checked (confirmed) against traceable lengths (NIST, PTB, NPL)**
  - Bars calibrated with interferometer at reference temperature
  - Fixed Targets on Bars
  - Bar Material = Object Material
  - Length uncertainty set by lab
Propagation of Uncertainty

- Effect of variable uncertainties (or errors) on the function uncertainty
- Probable true value lies in interval
  - $x - \Delta x$
  - $x + \Delta x$
- Define uncertainty by relative error $\Delta x / x$ (percentage)
- Assume difference between a measured value and true value is normally distributed using standard deviation as uncertainty of measurement
Input Component Uncertainties...

- **Measurement uncertainty is higher**
  - CTE uncertainty characterization is significant (> 5% of CTE)
  - Material temperature measurement uncertainty
  - Object temperature measurements one or a few observations

- **Survey scale is set with uncertain temperature and CTE**

- **Process time is lost**
  - Get measurements of scale bars to check to tight tolerances ...
    - re-measure bars n times → find right temperature
  - **Setting scale with less precise process then having to check with higher precision**
CTE Thermal Length Uncertainty

\[ L_i = L_0 (1 - \alpha \Delta T) \]
\[ f(L_0, \alpha, \Delta T) = L_i \]

Uncertainty of \( L_i \) is a function of \( L_0, \sigma_L, \alpha, \sigma_\alpha, \Delta T, \sigma_T \)

Example: 2 meter Alum Scale Bar from 10° to 40° C

\( L_0 = 2000 \text{ mm} \) \( \sigma_L = 0.02 \text{ mm} \)
\( \Delta T = -10 \ldots 20 \text{ °C} \) \( \sigma_T = 0.5 \text{ °C} \)
\( \alpha_{\text{alum}} = 23.8 \text{ ppm/°C} \) \( \sigma_\alpha = 5\% \alpha \)
Propagation of Uncertainty

Model: \( L_i = L_0(1 - \alpha \Delta T) = f(L_i) \)

\[
U[f] = S_f = \sqrt{\left( \frac{\delta f}{\delta L_0} \right)^2 \sigma^2_L + \left( \frac{\delta f}{\delta \alpha} \right)^2 \sigma^2_\alpha + \left( \frac{\delta f}{\delta \Delta T} \right)^2 \sigma^2_T}
\]

\[
U[L_i] = S_L = \sqrt{(1 - \alpha \Delta T)^2 \sigma^2_L + (L_0 \Delta T)^2 \sigma^2_\alpha + (L_0 \alpha)^2 \sigma^2_T}
\]

- Formula for the variance between products
- Propagation of error approach combines estimates from individual auxiliary measurements


Integrating Scale Bars/Lengths to Strengthen Metrology Network Uncertainty
Scale Bar Length Uncertainty vs. Temperature

2-m Alum Scale Bar Length Uncertainty (2-sigma) vs. Object Temperature

![Graph showing the relationship between measured scale bar temperature and length uncertainty.](image-url)
Scale Length Uncertainty Components

CTE Scaling Components of Unit Vector

Material Temperature (°C)

Component Uncertainty (Unit Vector)

Total Scale Uncertainty (mm)

Base Length Uncertainty
Temperature Uncertainty
CTE Uncertainty

Integrating Scale Bars/Lengths to Strengthen Metrology Network Uncertainty
Solution ... Better Metrology Practice

- Use traceable scale lengths to set object scale
  - Certified scale lengths of like kind material (soaked with object)
  - Multiple scale bar positions
  - Local Scale Differences
  - Observations from multiple stations
- Solve with Weighted Mean Scaling (Least Squares) per station/instrument
- Check scale with temperature and CTE
  - Confirm the scale factor maps to object temperature and material CTE
Scale Length in USMN

Integrate scale length into Uncertainty Field Analysis

- Scale length uncertainty from traceable certification
- Multiple bar positions and orientations
- Multiple stations
- Local scale deformations due to object temperature gradient
Scale with Certified Lengths...

- **4 Station Survey**
  - Alum 2.44 m Scale Bar in 10 positions
  - Temp: 23±0.5°C
  - Max Dist: 9.78 m
- **Potential Error with CTE Scaling ±0.13 mm**
  - @22.5 → -0.13 mm
  - @23.5 → +0.13 mm
- **Potential Error with Certified Length Scaling ±0.02**
- **Net Difference: 0.11 mm (≈ ±0.004” in 386”)**
Application: Thermal Comp Uncertainty on 9.78-m Aluminum Object

9.78-m Alum Object Length Uncertainty due to Thermal Comp (2-sigma) vs. Object Temperature

0.1 mm = 0.004”
9.78 m = 32 ft

Integrating Scale Bars/Lengths to Strengthen Metrology Network Uncertainty
Monte-Carlo Uncertainty Analysis

- Uncertainty Field Analysis includes scale bar constraints
  - Report Only
  - As Constraints
    - Weighted based on length and published uncertainty from lab

- Monte-Carlo Uncertainty Analysis and Validation for network with modeled scale bar constraints...
  - Confirm an instruments or stations performance within a network against certified length
  - Network Target field uncertainty analysis with graphical and component output

- Automated outlier characterization (Ranking %) and possible elimination

- Shop floor users to consistently and effectively use this advanced optimization technique
Target Uncertainty Analysis w/ Certified Lengths

Integrating Scale Bars/Lengths to Strengthen Metrology Network Uncertainty
Conclusions ... Summary

- Scaling 3D metrology surveys with CTE and object material temperature delta increases uncertainty
- Scaling with certified lengths reduces measurement uncertainty
  - Reduced Uncertainty
  - Enhance Traceable Reporting
- Certified Length Standard are weighted in network optimization
- Instruments Uncertainty Analysis and Reports are against traceable length standards
- Target Uncertainty Field Analysis includes traceable length standards
Integrating Scale Bars/Lengths to Strengthen Metrology Network Uncertainty

Questions

Scott Sandwith
Dr. Joe Calkins