Determining laser tracker alignment errors using a network measurement

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Abstract

A laser tracker is a coordinate measuring system that tracks a moving target reflector and measures the position of the target in spherical coordinates (d, θ, ϕ). The radial distance, or range component, d, is typically measured by an interferometer (IFM) or an absolute distance meter (ADM). The IFM or ADM laser beam is steered to track the moving target by a motorised gimbal mechanism. Angle encoders on the mechanism provide azimuth, θ, and elevation angles, ϕ, to the target.

The beam steering mechanism within the laser tracker is subject to misalignments, offsets and eccentricities that lead to errors in the measured coordinates. For this reason, all laser tracker manufacturers provide a means for correction in software of these systematic effects. The correction software relies on a model that describes the beam steering mechanism and its errors. The parameters of the model are usually derived from a combination of factory calibration and simple procedures that can be performed by the instrument user on the shop floor. The trade-off between time and complexity means that end-user procedures are not able to capture all the errors in the tracking mechanism simultaneously.

In this paper, we propose a simple test that is able to determine all the alignment and angle encoder errors of a laser tracker and their corresponding uncertainties. The test requires no specialist equipment and can be performed by an end-user in less than thirty minutes.

Keywords: laser tracker, alignment errors, network measurement.
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Introduction

A laser tracker is a high-accuracy, large volume coordinate measurement system that tracks and measures the spherical polar coordinates of a moving reflective target probe. The laser tracker comprises a laser interferometer (IFM) or absolute distance meter (ADM) for radial range measurements and two angle sensors. The measurement beam of the IFM or ADM is directed at all times to follow the target by a beam-steering mechanism operating in two orthogonal axes – azimuth and elevation. Angle encoders on each axis determine the instantaneous pointing direction of the beam. This combination of sensors directly measures the spherical polar coordinates, (R, θ, ϕ), of the target.

INSERT Fig 1

Figure 1 Schematic representation of an *idealised* laser tracker.

In an idealised laser tracker, as shown schematically in Figure 1, the two rotation axes – standing axis and transit axis - would be orthogonal and intersect at a point. The measurement laser beam would originate from the point of intersection of the axes and be aligned normal to the transit axis. The rotary encoders would be coaxial with the rotation axes and be free from scale and alignment errors.

In reality, a laser tracker is subject to imperfections in the form of misalignments, eccentricities and offsets that give rise to systematic errors in the measured coordinates of the target if the observed sensor readings are used directly. In practice, corrections have to be applied to the observed sensor readings to account for these misalignments. These corrections are based on a combination of manufacturer calibrations, instrument self-checks during start-up and tests by the operator.

INSERT Fig 2

Figure 2 Schematic representation of a laser tracker with alignment errors.

Over recent years documentary standards have been developed that are aimed specifically at testing the performance of laser trackers with a view to identifying faults, the effect of miss-calibration or poor correction of systematic errors. The ASME B89.4.19[i] standard prescribes a number of tests of the performance of laser trackers including a set of length measurements of a reference artefact and two-face tests. More recently, the draft VDI/VDE 2716[ii] and draft ISO 10360 part 10[iii] documents propose a similar set of tests. Loser[iv] compares these standards and describes a method of realising reference lengths suitable for the tests described. These standards test the length measurement capability of the tracker against the manufacturer's maximum permissible error (MPE) rather than attempt to determine instrument accuracy. Sandwith and Lott[v] have proposed a test based on the Unified Spatial Metrology Network (USMN) feature within SpatialAnalyzer[vi]. This test uses measurements of a network of fixed points from a number of different instrument locations and bundles the data together to calculate the overall uncertainty of the individual sensor readings – range and two angles. This test gives a single uncertainty value for each sensor.

To correct sensor readings for mechanical offsets within the mechanism, it is necessary to understand how these offsets and alignments affect the individual sensor readings. Loser and Kyle[vii] have described the misalignments for a laser tracker design that uses a gimballed mirror for beam-steering. The fifteen parameters of their model are broadly categorized as offsets, tilt deviations and angle encoder parameters. Loser and Kyle derived formulae to correct the observed sensor readings for the influence of these error parameters and describe experimental setups – five in total – that can be deployed to measure the error parameters. More recently, Muralikrishnan *et al*[viii] at the National Institute of Standards and Technology (NIST) have modified the Loser and Kyle error model to reflect a slightly different mechanical arrangement within the tracker mechanism – that of a tracker with the beam source on the rotating head rather than a gimballed mirror. They also derive the sensitivity of the length measurement and two-face tests described by the ASME B89.4.19 standard to the individual error parameters and show that some parameters are not detectable by these tests.

Laser tracker manufacturers provide users with software-assisted methods for determining some of the parameters and updating the parameter file used by the instrument to correct measurement data. However, users are not typically provided with methods for determining the errors associated with the angle encoders.

Gassner and Ruland[ix] developed a method for calibration of the azimuth angle encoder based on back-to-back calibration against a rotary table. The rotary table was calibrated using a polygon and autocollimator. For practical reasons, this approach was not suited to elevation angle calibration. More recently Muralikrishnan *et al*[x] have developed a method of calibrating the low-order harmonic errors of the azimuth encoder using an un-calibrated, but stable length artefact. They demonstrate a standard uncertainty of 0.4 µrad.

In this paper, we describe a variant of the NIST model and a simple test that can be used to determine ***all*** the error parameters described in the model described below, including the angle encoder errors. The method involves mathematically fitting measurements of a network of fixed points to a mathematical model describing the setup and the errors of the instrument The key features of the technique are that:

- little or no specialist hardware is required, so the technique is applicable by end users;
- the test uses both front-face and back-face measurements for greater sensitivity to the errors;
- the IFM (if fitted) can be used for improved accuracy;
- all parameters are determined simultaneously;
- random noise effects are averaged over many samples, and
- the uncertainties in the measured error parameters are calculated using statistically rigorous methods.

The approach described here is analogous to that in determining the parametric errors (scale, straightness, roll, pitch and yaw) associated with a CMM. A set of experiments can be set up to measure each of the error sources individually. Alternatively, measurements of a ball plate in a number of positions can be made and all the parametric errors determined from a model fit to the observed coordinate data[xi]. Similarly the parametric errors can be determined from a model fit to Laser Tracer measurements[xii].

The NPL Tracker Error Model

The NPL model described below is based very closely on that proposed in the NIST paper[viii], but with some differences. For comparison purposes, the error parameters used by the NIST model have been cross-reference with those of the NPL model in Table 1.

A brief description of each error follows. For a detailed explanation of these errors the reader is directed to the NIST paper.

Error description	NIST parameter	NPL parameter
Beam transit offset	x_{1t}	$-h_2$
Beam Z offset	x_{1m}	$-h_9$
Transit offset	x_2	$-h_1$
Vertical Index offset	x_3	$-h_{10}$
Beam Tilt	x_{4t}	$-h_3$
Transit Tilt	x_5	$-h_4$
Encoder Eccentricity	x_{6x}	$-h_5$
“	x_{6y}	$-h_6$
“	x_{7n}	$-h_{11}$
“	x_{7z}	$-h_{12}$
Birdbath Error	x_8	$-\lambda$
Encoder scale	x_{9a}	$-h_7$
“	x_{9b}	$-h_8$
“	x_{10a}	$-h_{13}$
“	x_{10b}	$-h_{14}$

Table 1 Laser tracker error model parameters.

The error parameters can be broken down into groups associated with alignment of the axes, the alignment of the laser beam, and errors associated with the encoders. It is interesting to note that typical operator test procedures prescribed by the tracker manufacturers do not determine angle encoder errors; these are presumed to be fixed until the next factory calibration.

Axis alignment errors

Transit offset

The transit offset, h_1, is defined as the (minimum) separation distance between the standing and transit axes.

Transit tilt

Transit tilt, h_4, is the squareness error between the two axes.

Laser beam alignment errors

Beam offsets

Parameters h_2 and h_9 describe displacements of the laser beam origin along the transit axis and an axis normal to the beam direction, respectively.

Beam tilt

This parameter, h_3, is a measure of the angle between the laser beam and the transit axis.

Vertical index offset

The vertical index offset, h_{10}, is an offset in the vertical angle encoder when the beam is directed normal to the standing axes. It is indistinguishable from beam tilt in the same direction so is included as a beam misalignment.

Birdbath error

Though not strictly a beam alignment error, the birdbath error has been included here as it is related to the measurement laser beam. The birdbath error, λ, is an error in the calibration of the distance to the fixed reference point on the tracker used to set the interferometer datum.

Angle encoder errors

Encoder eccentricity

These parameters describe the misalignment of the angle encoders with respect to the rotary axes. h_5 and h_6 are misalignments of the azimuth encoder, and h_{11} and h_{12} are misalignments of the elevation encoder.

Encoder scale

These are second order harmonic errors in the encoders.

These parameters have been incorporated into a mathematical mode that describes the effect of the errors on the measured parameters. The NIST error model (that is applicable to front-face measurements only) is:

$$R_c = R_m + x_2 \sin V_m + x_8$$

$$H_c = H_m + \frac{x_{1t}}{R_m \sin V_m} + \frac{x_{4t}}{\sin V_m} + \frac{x_5}{\tan V_m} + x_{6x} \cos H_m - x_{6y} \sin H_m + x_{9a} \sin 2H_m + x_{9b} \cos 2H_m$$

$$V_c = V_m - \frac{x_{1m}}{R_m} + \frac{x_2 \cos V_m}{R_m} + x_3 + x_{7n} \cos V_m - x_{7z} \sin V_m + x_{10a} \sin 2V_m - x_{10b} \cos 2V_m$$

where, R_c, H_c and V_c are the corrected range, azimuth and elevation readings and R_m, H_m and V_m are the un-corrected range, azimuth and elevation readings and the parameters, x_i, are the instrument error parameters listed in Table 1.

This model has been modified in the following ways:

1. Re-defined the elevation angle zero position to be that when the laser beam is horizontal rather than vertical.
2. Extended the model to include back-face measurements.
3. Introduced noise terms on all the measured data, ε_D, ε_A and ε_E. These are samples from a statistical distribution with an expectation of zero and standard deviations of σ_D, σ_A and σ_E respectively.

4. Introduced a scale error, μ, on the range measurement.

The NPL model is:

$$d = (1+\mu)d^* + \lambda + h_1 \cos\phi + \varepsilon_D$$

$$\theta = \theta^* + \frac{h_2}{d\cos\phi} + \frac{h_3}{\cos\phi} + h_4 \tan\phi + h_5 \cos\theta - h_6 \sin\theta + h_7 \sin 2\theta + h_8 \cos 2\theta + \varepsilon_A$$

$$\phi = \phi^* + \Gamma\left(-\frac{h_9}{d} + h_{10} + h_{11}\sin\phi - h_{12}\cos\phi + h_{13}\sin 2\phi\right) + \frac{h_1 \sin\phi}{d} - h_{14}\cos 2\phi + \varepsilon_E$$

where d, θ and ϕ are observed values and d^*, θ^* and ϕ^* are modelled values of the measurands based on a ideal tracker geometry. In this way, the observed values of the sensor readings are regarded as perturbations of the modelled values taking into account the geometric errors represented by **h**. The parameter θ in the NPL model is equal to V_m in the NIST model, and ϕ in the NPL model is equal to (π-V_h) in the NIST model. The parameter Γ is introduced to account for sign changes in the parameters during back-face measurements. It takes the value 1 for front-face measurements and –1 for back-face measurements. In front-face mode, the observed angles in the equations above are as reported by the tracker. In back-face mode, the angles reported by the tracker, θ_R and ϕ_R, are reported as though the tracker were in front-face mode, so the reported back-face angles are modified thus:

$$\phi = (\pi - \phi_R)$$
$$\theta = (\theta_R + \pi)$$

(Note that we have the <u>observed</u> values on the left hand side whereas the NIST model has the <u>true</u> values on the left - the signs of the parameters are therefore different between the models, hence the negative signs in Table 1.) By re-arranging the equations in this way with the modelled values on one side of the equations and the observed values on the other, the least squares model fit determines the maximum likelihood estimate of the model parameters and valid uncertainty estimates can be made following standard statistical modelling methods.

It is interesting to note the potential for strong correlation between the birdbath distance, λ, and the transit offset, h_1, in the first equation. These parameters become inseparable when the vertical angle is zero (laser beam horizontal). Standard birdbath tests, which measure the distance between two points from inside and outside the line joining the two points, only work correctly if the transit offset has already been determined. This correlation requires specific action to be taken during the measurements to weaken the correlation thereby allowing each parameter to be evaluated.

Experimental Setup and Procedure

A key requirement when developing this method was that the process should be easy to set up. We therefore aimed for a network setup that could be constructed in the corner of a room, utilising the walls as relatively stable locations to fix target nests. In our laboratory, solid brick and granite benches are positioned along the walls. These were used to support the tracker and some target nests. The network used in shown diagrammatically in Figure 3.

The arrangements of target points and tracker positions were chosen such that the following criteria were fulfilled:

- The whole angular range of the tracker was exercised in azimuth and elevation.
- Target to tracker range varied from <0.2 m to approximately 6 m.
- Two birdbath test-like setups were arranged (indicated by the red lines in Figure 3); one with the two target points horizontal and one with the target points aligned at approximately 45 ° to the vertical.

INSERT Fig 3

Figure 3 The network arrangement showing the sixteen target positions and four tracker positions used during the experiment. Note the two birdbath test-like lines highlighted in red.

The first two criteria ensure that data is obtained over a reasonable working volume. The third criterion is a deliberate ploy to break the correlation in the model between the birdbath distance and the transit offset, described above.

INSERT Fig 4

Figure 4 This plot shows the spread of all the point relative to the tracker. The four different colours represent points measured from the four different tracker locations.

Prior to making the measurements, the tracker was calibrated using the manufacturer prescribed methods. These tests determined all error parameters apart from those related to the angle encoders. All the parameters determined by the manufacturer procedure were noted for future reference. The parameter file was then manually edited to set all correction parameters to zero and uploaded onto the tracker controller. All subsequent measurements were thus uncorrected and subject to all error sources. The network was then measured from four different tracker positions.
All measurements were made in IFM mode. All visible targets were measured from each tracker location (not all targets were visible from all locations). All targets were measured with a spherically mounted retro-reflector (SMR) in front-face mode first, with the SMR returned to the birdbath after any beam break. All visible targets were then measured in back-face mode.

Data Fitting

Estimates of the model parameters were determined using a least squares parameter estimation approach[xiii]. The parameters in the model are the target locations $\mathbf{x}_I = \{\mathbf{x}_i\}$, and the configuration **b** comprising the location and orientation of the trackers, the birdbath error, the scale corrections and the geometric errors **h**. These parameters are determined by minimising an objective function of the form

$$F(\mathbf{x}_I, \mathbf{b}) = \left(\sum_j \mathbf{f}_j^{\mathrm{T}}(\mathbf{x}_j, \mathbf{b}) \mathbf{f}_j(\mathbf{x}_j, \mathbf{b}) \right) + \mathbf{f}_0^{\mathrm{T}}(\mathbf{b}) \mathbf{f}_0(\mathbf{b})$$

subject to constraints $C^{\mathrm{T}}\mathbf{b} = \mathbf{b}_0$.

The first term on the right of the equation above represents the miss-match between predicted sensor measurements and their observed values while the second term can be used to incorporate any prior information about the configuration parameters, in particular, the length and scale offsets. The constraints are used to fix the frame of reference of the laser trackers. These constraints are implemented in such a way as to equalise, approximately, the uncertainty in laser tracker positions across all tracker positions with the benefit that the uncertainties associated with the computed target estimates have only a modest component arising from the uncertainty in position[xiv].

The fact that each summand function involves at most one set of target co-ordinates allows the optimisation to be performed efficiently[xv,xvi]

The summand functions in the objective function incorporate weights given by the inverse of the estimate of the standard deviation of the random effects associated with each sensor measurement. (Assigning the weights in this way guarantees that the nonlinear least squares solution is the maximum likelihood estimate of the parameters.) At the solution, the variance matrix associated from the fitted parameters can be determined from the Jacobian matrix of partial derivatives of the summand functions with respect to the parameters[xvii,xiii] using a propagation of uncertainty approach. The standard uncertainties associated with the fitted parameters are given by the square root of the diagonal elements of the variance matrix. These uncertainty estimates are based on the input uncertainties, σ_D, σ_A and σ_E, associated with the random effects for the sensor measurements. Based on the value of the objective function $F(\mathbf{x}_I, \mathbf{b})$ at the solution, all the

input uncertainties can be rescaled using a single scale parameter so that the observed variation in the residuals matches that predicted by the input uncertainties - this procedure is sometimes known as the Birge correction.) A more flexible adjustment to the input uncertainties can be achieved by using separate scale corrections for the displacement and angle measurements and has been implemented using a Bayesian methodology in which prior distributions for the input uncertainties are updated on the basis of the observed data[xviii].

Experimental Results

A comparison between the results obtained with the manufacturer prescribed tests and the new test is shown in Table 2. The standard uncertainties ($k = 1$) associated with our results are included in parentheses.

Error description	NPL Parameter	Standard Test	NPL Test
Beam transit offset	h_2	-90 µm	-86.4 µm (1.9 µm)
Beam Z offset	h_9	-7.0 µm	-4.0 µm (1.8 µm)
Transit offset	h_1	11.9 µm	-3.8 µm (0.34 µm)
Vertical Index offset	h_{10}	-11.1 µrad	-0.5 µrad (1.0 µrad)
Beam Tilt	h_3	-38.9 µrad	-45.3 µrad (1.1 µrad)
Transit Tilt (squareness)	h_4	-87.7 µrad	83.9 µrad (1.1 µrad)
Birdbath Error	λ	-2435.1µm	–2444.1 µm (1.1 µm)

Table 2 Comparison of parameters measured using the manufacturer's standard test and the new NPL proposed test. The uncertainties associated with the NPL results are in parentheses. (Note that sign conventions may be different between the two sets of parameters)

INSERT Fig 5

Figure 5 Comparison between manufacturer's azimuth encoder correction table (blue line) and that generated from the parameters of our model (green line).

For the laser tracker tested, the angle encoder errors were factory calibrated and encoded within a lookup table in the parameter file. In order to compare results obtained using the proposed method with those obtained by the manufacturer's prescribed methods, the error parameters that were obtained from the fit of the data to the model were used to generate a lookup table. The new lookup table was then compared it to that stored in the parameter file. The results for both angle encoders are shown in Figures 5 and 6.

INSERT Fig 6

Figure 6 Comparison between manufacturer's elevation encoder correction table (blue line) and that generated from the parameters of our model (green line).

Discussion

An important consideration when determining parameters by least squares fit is that of correlation between the parameters. A strong correlation between two parameters means that, as far as the observed data is concerned, changing the value of one parameter can be compensated by changing the value of the other by an appropriate amount. If the observed data is representative of all future data, such correlation is benign because it will have no observable effect. However if the data associated with the calibration is a small subset of possible future uses of the tracker, then the error correction determined by the calibrated values of the model parameters could still leave the tracker performance with significant systematic effects.

Figure 7 shows the graphical representation of the correlation matrix associated with the parameter values presented above. The dark red colour indicates a stronger correlation. A correlation coefficient of 0.9 is larger than desired but not necessarily unacceptable.

It is interesting to compare the results obtained using the new test with those obtained using conventional methods in the context of the correlation information. In general there is good agreement between conventional methods and the new technique for the parameters listed in Table 2 with the exception of λ, h_1 and h_{10}. Parameters λ and h_1 had been previously identified as potentially correlated and this issue was addressed in the experimental setup. Note that the analysis reported here shows no significant correlation between these two parameter for the results obtained with the new method. Nevertheless there are significant differences between the results obtained with the two methods. However, the sum of the absolute values of these two parameters agree between the two measurement techniques to less than 1µm.

INSERT Fig 7

Figure 7 Representation of the correlation between the fitted parameters. The numbers on both axes represent the index, *i*, of the parameter h_i. The zeroth element represents the parameter λ.

The plot of Figure 5 shows apparently good agreement on the azimuth angle errors, although the conventional calibration shows signs of higher order harmonics that we have not modelled.

The agreement between the two methods for the elevation angle encoder is not so good. The calculated uncertainty (see error bars in Figures 5 and 6) for the technique discussed here is higher than for the azimuth parameters and the plot of Figure 7 suggests strong correlation between parameters h_{12} *and* h_{14}; both are associated with the elevation encoder. Another parameter, h_{10}, is also in poor agreement with the conventionally determined result, but this parameter appears to be correlated with h_9.

Another correlation that we observe is between parameters h_2 and h_3. Close examination of the equation for azimuth angle above suggests that this correlation could be weakened by making measurements over a larger distance range, *d*.

These initial results indicate that the method shows strong promise, but that further work on the experimental design is required to address correlations.

Conclusions

It has been shown that it is possible to determine the geometric errors of a laser tracker from a survey of targets from a small number of tracker positions. The experimental strategy involves no specialist equipment and the calibration experiment can be performed by the majority of users. The approach depends on software to analyse the observed data to determine the error sources. The model-fitting approach reflects the systematic and random error sources, delivering a variance matrix for the fitted parameters. This variance matrix can be used to determine the uncertainty contribution from the geometric errors to the uncertainty in the target locations when the calibrated tracker is in use. The use of backface as well as frontface measurements is beneficial in reducing correlations amongst the fitted parameters leading to a more accurate determination of the geometric error parameters. For a well-designed calibration experiment, the uncertainty contribution arising from the (corrected) geometric errors will be small so that the uncertainties associated with the target locations are dominated by the random effects. The ability to take into account calibration uncertainties is a necessary step for incorporating laser tracker measurements into a framework of traceable dimensional metrology.

Future Work

The next logical step for this work is to optimise the positions of the target and tracker points to reduce correlations and improve accuracy. Then to compare the performance of the tracker when using the error correction parameters determined using the new technique with its performance when using the parameters determined using conventional methods. This could be done using the length and two-face tests described in ASME B89.4.19. Following on from that, some possible future developments of this work will include:

- Optimisation of the network to reduce the number of points and therefore to reduce measurement time.
- Extension of the model to higher-order harmonic errors of the encoders.

- Inclusion of ADM scale and offset parameters in the model and adapt the test to determine these parameters.

An alternative modelling approach is also being developed by NPL. The model above is based on the NIST model in which the observed sensor measurements for a non-ideal geometry are corrected to simulate what the sensor measurements would have been if the geometry were perfect. The model being developed is derived directly from Figure 2 and reflects more closely the nonlinear nature of the error contributions and simultaneously deals with front and back face measurements.

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