



REPORT OF SPECIAL TEST

NIST Test No. 685-O-0000048577-23

Service ID/SKU# 10070C, 10072C (4)

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Item: SpatialAnalyzer - SA 2023.1

The least-squares fitting features of this software package was tested on 240 data sets, representing the following geometry types: lines, lines in 2D, circles, circles in 2D, planes, spheres, cylinders, and cones. The test procedures followed are documented in ASME B89.4.10-2000 and NISTIR 5686. The ordering of the data points in each data set was randomized. For the purposes of this testing, each data point was given equal weight in the least-squares fit objective.

The uncertainties associated with the reference values were evaluated following NIST Technical Note 1297, *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, which is considered to be part of this Report. The expanded uncertainty, U , is calculated using a coverage factor $k = 2$. For a measured value of length (or angular measure), L , the true length (or angular measure) is contained in the interval $[L - U, L + U]$ with a level of confidence of approximately 95 %. The results of the test are as follows:

ASME B89.4.10-2000 Standard Default Test

Geometry Type	Mean (RMS) Deviation			
	Separation (μm)	Tilt (arc seconds)	Radius/dist (μm)	Apex (arc seconds)
Lines	$< 10^{-5}$	$< 10^{-7}$	—	—
Lines 2D	$< 10^{-5}$	$< 10^{-7}$	—	—
Planes	3×10^{-5}	1.4×10^{-4}	—	—
Circles Full 3D	1.4×10^{-4}	6×10^{-4}	5×10^{-5}	—
Circles 2D	1.2×10^{-4}	4×10^{-4}	6×10^{-5}	—
Spheres	1.2×10^{-4}	—	3.4×10^{-5}	—
Cylinders	$< 10^{-5}$	1.5×10^{-7}	$< 10^{-5}$	—
Cones	$< 10^{-5}$	$< 10^{-7}$	$< 10^{-5}$	$< 10^{-7}$

In each of the results above, the NIST ($k = 2$) expanded uncertainty, U , is less than 1.0×10^{-8} μm or arc seconds, as applicable. This is due in part to the fact that the NIST reference results were calculated using precision that is better than double precision computations.

Data sets for the categories “Lines 2D” and “Circles 2D” are restricted to two-dimensions. That is, all the data points lie within a plane, and that plane is parallel to one of the x - y , y - z , or z - x coordinate planes. Data points for all other geometries are not restricted to any plane.

Two possible fit objectives exist for least-squares 3D circle fitting. The fit objective for the case of 3D circles in this test was to minimize the sum-of-squares of the full-3D distances from the points to the circle. The alternative fit objective is to fit a least-squares plane to the data points, then project the data points into that plane, and then fit a least-squares 2D circle to the projected data within that plane.

The test conditions, particularly the specifications for the test data sets, comply with the default test specified in Standard ASME B89.4.10-2000 (Reference 1). Some conditions are summarized as:

Sampling strategy	Points were regularly spaced over the sampling region but in a randomized order with the exception of the first three points in each data set for cylinders and cones, as noted above.
Measurement error	Uniformly random measurement error simulations were included.
Form errors	Several errors specified in the standard, including bends, sinusoidal, step errors, tapers, etc.
Range of part size	1 mm to 500 mm.
Part origin	Within 1000 mm of coordinate system origin.
Aspect ratios	Planes: maximum length:width ratio was 50. Cylinders and Cones: aspect ratios between 0.02 and 10.
Partial features	Circles: arcs as small as 90° . Spheres: hemispheres, 90° polar patches, and 30° bands. Cylinders and Cones: 90° to 360° sweeps.

For applications of the software that are within the scope of the test conditions described above, the root mean square (RMS) value given in the table of results is a reasonable evaluation of the fitting software’s standard uncertainty contribution to the uncertainty of a corresponding measurement.

micrometers for lengths and in arc seconds for angles. The values reported in this Report of Special Test apply only to the software tested and only in the computing environment in which it was tested. NIST cannot guarantee that the user’s software will have the same value as reported by NIST when used in another facility at a later date.

This Special Test was carried out as follows: NIST generated data sets simulating the ranges of test conditions described above in accordance with the ASME B89.4.10-2000 Standard. NIST also generated reference fit results using NIST’s Algorithm Testing System internal algorithms. The reference results are traceable to the same units of the data sets, specified in the Standard. The customer received the NIST-generated data sets in ASCII format and generated corresponding fit results using the software under test. NIST then compared each of the customer’s fits to the reference fit for the corresponding data set using procedures set forth in the Standard. The reported test results for each geometry type are the RMS value deviations between the customer’s fits and the reference fits for all data sets corresponding to that geometry type. According to the Standard, when deviation results are less than 10^{-5} μm or 10^{-7} arc seconds, these values are reported as “ $< 10^{-5}$ ” and “ $< 10^{-7}$.”

The following table displays the maximum observed error (deviation) of each evaluation parameter for each geometric feature type.

Geometry Type	Maximum Observed Deviations					
	Separation (μm)	Tilt (arc seconds)	Radius/dist under (μm)	Radius/dist over (μm)	Apex under (arc seconds)	Apex over (arc seconds)
Lines	$< 10^{-5}$	1.3×10^{-7} data set 26	————	————	————	————
Lines 2D	$< 10^{-5}$	1.2×10^{-7} data set 15	————	————	————	————
Planes	1.6×10^{-4} data set 28	5.7×10^{-4} data set 9	————	————	————	————
Circles Full 3D	6.6×10^{-4} data set 13	3.0×10^{-3} data set 13	2.8×10^{-5} data set 12	2.6×10^{-4} data set 22	————	————
Circles 2D	4.7×10^{-4} data set 14	1.7×10^{-3} data set 14	3.0×10^{-4} data set 14	1.2×10^{-4} data set 27	————	————
Spheres	3.4×10^{-4} data set 27	————	1.0×10^{-4} data set 14	8.8×10^{-5} data set 6	————	————
Cylinders	$< 10^{-5}$	5.1×10^{-7} data set 17	$< 10^{-5}$	$< 10^{-5}$	————	————
Cones	$< 10^{-5}$	1.4×10^{-7} data set 1	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-7}$	$< 10^{-7}$

Detailed data concerning this Special Test are available from NIST on request. For detailed descriptions of the technical approach used for these test services and specifics on the test procedures see the following references.

- [1] ASME B89.4.10-2000, *Methods for Performance Evaluation of Coordinate Measuring System Software*, B89.4.10, American Society of Mechanical Engineers, New York, NY, 2000.
- [2] Diaz, C., *Algorithm Testing and Evaluation Program for Coordinate Measuring Systems: Testing Methods*, NISTIR 5686, National Institute of Standards and Technology, Gaithersburg, MD, 1995.
- [3] Diaz, C., and Hopp, T., *Evaluation of Software for Coordinate Measuring Systems*, proceedings of the 1995 SME Clinic, CMMs Week, June 5-8, Society of Manufacturing Engineers, Dearborn, MI; also in Proceedings of the 1995 Interface Symposium, June 21-24, Interface Foundation of North America, Carnegie Mellon University, Pittsburgh, PA.
- [4] Hopp, T. and Levenson, M., "Performance Measures for Geometric Fitting in the NIST Algorithm Testing and Evaluation Program for Coordinate Measuring Systems," *NIST Journal of Research*, **100** (5):563-574, 1995.
- [5] Rosenfeld, D., *User's Guide for the Algorithm Testing System Version 2*, NISTIR 5674, National Institute of Standards and Technology, Gaithersburg, MD, 1995.
- [6] Rosenfeld, D., *Reference Manual for the Algorithm Testing System Version 2*, NISTIR 5722, National Institute of Standards and Technology, Gaithersburg, MD, 1995.
- [7] Shakarji, C.M., *Least Squares Fitting Algorithms of the NIST Algorithm Testing System*, *Journal of Research of the National Institute of Standards and Technology* **103** (6), 633-641, 1998.
- [8] Taylor, B. N. and Kuyatt, C. E., *Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results*, NIST Technical Note 1297, National Institute of Standards and Technology, Gaithersburg, MD, 1994.

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Test System: NIST ATS Version 2.0 on Windows 10 Enterprise, Intel Core i5-10310U
CPU @ 1.70 GHz 2.21 GHz

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