Real-time 5-Micron Uncertainty with Laser Tracking Interferometer Systems using Weighted Trilateration

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Abstract

Weighted trilateration numerical methods can improve measurement uncertainties with commercial Laser Tracking Interferometer systems on large parts to 5 microns (1-sigma). Weighted trilateration calculations use distance and angle measurements to determine the three-dimensional coordinates of unknown positions. These calculations give higher weight to the interferometer distance measurement from laser trackers; verses the weight given to the laser tracker's angle measurements. This solution technique effectively provides a more accurate measurement for several reasons, it optimizes the instruments strengths, it uses the extra information (i.e., angles) to strengthen the solution, and it provides for robust error checking. Laser Tracker Trilateration positioning systems can be used in applications where the use of other positioning systems is impractical. Since the measurement instruments are portable and are able to track and report results in real-time, they can be used in virtually any location to provide very accurate large-scale measurements of dynamic events. This presentation discusses and illustrates the weighted trilateration mathematical solution, test results from simulated datasets, and from shop floor experiments with three and four simultaneously connected laser trackers using the SpatialAnalyzer software platform.

Keywords: Trilateration, uncertainty, accuracy, 3D, point, distance, scaling, weighting, networks

Introduction

Trilateration is a general technique that uses at least 3 lengths[1] from known positions to solve for 3D point coordinates. In it simplest use, geometric principles are employed to find one location when given it's distance from a set of already known locations. By constructing a series of triangles adjacent to one another, a surveyor can obtain the other distances and angles that would not otherwise be measurable. Trilateration was little used in comparison to triangulation. However with the development of tracking interferometer distance-measuring devices and computing platforms that can integrate the measurements from at least three of the systems in real-time, trilateration can become a common and preferred system.

The technique can be extended (i.e., generalized) to solve for the positions of the measurement devices and a set of common points were the distance between the measurement devices and points is known.[2] If the measurement device is also able to measure the horizontal and vertical angles between the instrument and point, these measured attributes can also be included in the solution to further strengthen the numerical rigor and rate of convergence to the most probable values, for the points and instrument coordinates.

The trilateration technique can be implemented in a numerical method known as a bundle adjustment. A bundle adjustment is a well-understood method for reliably and consistently computing point and instrument locations by integrating a consistent model with weighted measurements [1,2]. It is particularly useful when more that the minimum amount of data is collected. It maximizes the accuracy by allowing each individual measurement to be weighted based upon its type and accuracy [3]. This presentation discusses and illustrates the weighted trilateration mathematical solution, test results from simulated

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1 Only 3 lengths are required if it is known which side of the base triangle the measurements were made. The ambiguity results from the solution being valid on both sides of the base of the pyramid. A unique solution is available with 4 lengths from known locations.

2 If 4 distance measuring instruments are used, distance measurements to at least 17 points are needed to solve for the relative locations of each 3D point and instrument in X, Y, Z, and Yaw, Pitch and Roll.

datasets, and from shop floor experiments with three and four simultaneously connected laser trackers using the SpatialAnalyzer software platform.

Weighted Trilateration Mathematical Solution

Bundle Adjustment

A bundle adjustment is a numerical algorithm used to refine redundant measurements from triangulation, spherical measurement systems or other dimensional measurement system into the best possible point coordinates. A bundle adjustment is a standard photogrammetric technique for optimizing the 3D location of a set of points from multiple images. The concept of bundling has been extended to include computer-aided theodolites and then to include laser-tracking interferometers [1].

The technique assumes redundant measurements are available, and then it optimizes the possible instrument and point positions in which the sum of the squares of the measurement residuals is minimized. It is typically a least-squares optimization mechanism for instruments and points. It adjusts the positions of the points and instruments until the sum of the squares of the differences between the measurements between the points and instruments are minimized. For a group of equally weighted observations (or measurements), the basic condition that is enforced in least squares adjustments is that the sum of the squares of the residuals is minimized [2]. Residual is another name for the difference between the measured value and its most probable value. So for 3D coordinate metrology, one example of a residual would be the difference between the true X-coordinate for a point and the actual computed X-coordinate measurement.

Weighting

There are several issues to consider when weighting the measurements to bundle adjust laser trackers observations. These control the importance of different parameters on the overall optimization process [1]. To have a well-balanced model, using realistic weights is essential. The constraints should be generally characterized by their standard deviations. In this application, the constraints also have to be presented to users in a fashion that can be easily understood and reliably implemented. Hard coding the weighting of the constraints can cause the model to be non–convergent, so the weight parameters have to be accessible to the user.

The relative accuracy of measurements used in the Bundle Adjustment is accounted for by weighting the more precise measurements higher than the less precise measurements. Traditionally laser tracker measurements were weighted statically, meaning each angle measurement was weighted the same. Similarly, each range measurement was weighted with the same value. It can be shown that the range and angle measurement accuracy are range dependent, and as such, a refinement to the weighting scheme could yield improved results [3].

Test Results

Simulated Datasets

Two set of simulated angles and ranges from 4 trackers locations were developed for the Large Millimeter Telescope Project. The data simulates the parabolic surface panel metrology aspects of the project. Each of the 180 panels for the 50 m diameter primary are about 3 by 5 m in size. The panels are almost flat with a radius of curvature greater that 35 m. A set of measurements is required on a 100 by 100 mm grid over the panel surface. The test objective is to analyze the properties of laser tracking metrology systems for surface curvature measurement and to study the projected uncertainties of a four-headed laser tracker measurement system.

The measurement sets were composed of four sets of angles and ranges from each tracker to each point. The first set was a "perfect set" in that it contained no simulated measurement error. The second set of angles and ranges included, "noise."

The simulated data was imported into the SpatialAnalyzer as measurements from 4 laser trackers, see Figure 1. The Leica tracker instrument was used in this simulation, but SMX or API would have served the same purpose. Figure 2 and Figure 3 show the instruments and points with their measurements.. Figure 2 shows the measurements from all 4 instruments, while Figure 3 only shows instrument one's measurements.

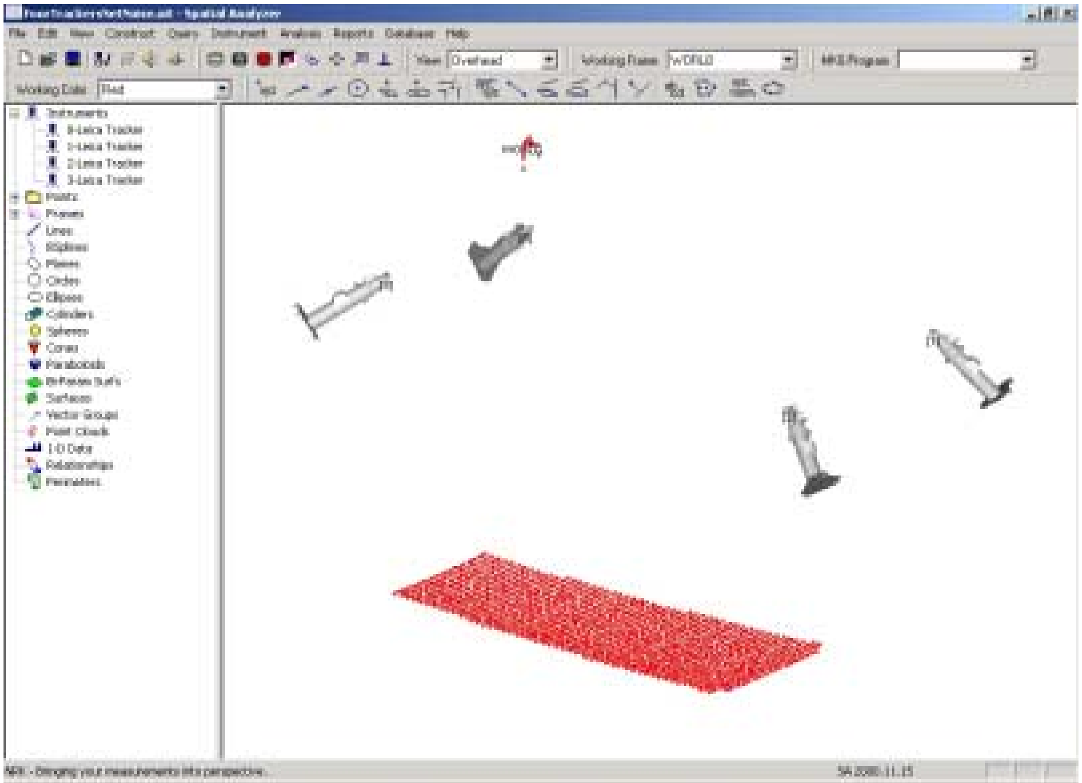

Figure 1: 4 Laser tracker Trilateration using simulated test data.

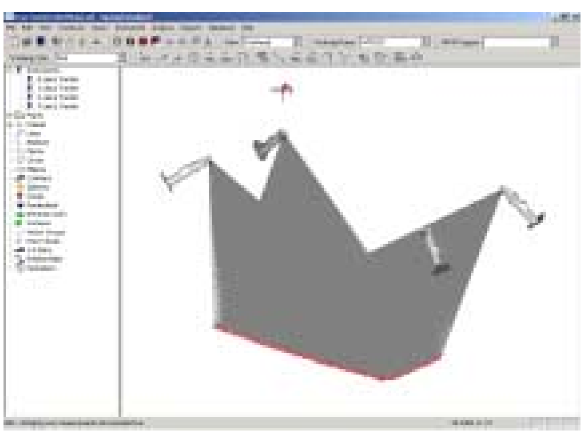

Figure 2: Trilateration setup showing measurements from all four trackers.

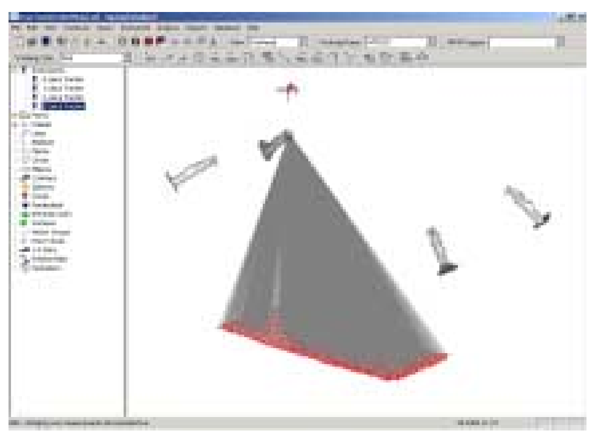

Figure 3: Trilateration setup, showing the measurements from instrument number 1 of 4.

The two-sigma measurement uncertainty values were set to 2 arc seconds for the angles and 2 parts per million for the ranges plus 2 microns offset in range[3]. This variation was induced onto the "Noise" data to simulate tracker inaccuracies as well as atmospheric variation effects. There were 5988 total measurements from the 4 trackers to 1497 targets in each measurement set.

[3] LMT/GTM Panel Metrology Report, by Read Predmore, dated 13, November, 2000

Both sets of data were bundled. Different uncertainty relationships between the relative angles and range uncertainty were processed. The parameters under which the test results were processed are presented in Table 1. The first run for both sets of data used equal weights the angles and ranges. Because the first dataset was perfect data (no noise) the subsequent runs were not done. Run 2 weighted the angles at 5 times less precise than their range counterparts, and Run 3 weighted the angles out of the solution. Analysis between the weighting schemes is presented in Figures 4 and 5.

Table 1: Relative weights and instrument uncertainty for the different datasets and runs.

		SET 1	SET 2		
Run		1	1	2	3
Relative Weights	Angles	1.0	1.0	0.2	0.0
	Ranges	1.0	1.0	1.0	1.0
Instrument Uncertainties	Angles Deg.	0.0002778	0.0002778	0.0002778	0.0002778
	Range Offset (mm)	0.002	0.002	0.002	0.002
	Range Dependence (ppm)	1	1	1	1

Comparisons of the results are shown in Tables 2 and 3. Table 2 presents a summary of the bundle residuals for each of the scenarios. These are summary statistics for the angles and range components of the measurements.

Table 2 : Bundle residual summary statistics comparing weighting schemes.

		SET 1	SET 2		
Run		1	1	2	3
RMS	Deg.	0.00000	0.00035	0.00040	0.00257
Max	Deg.	0.00001	0.00086	0.00084	0.00344
Avg	Deg.	0.00000	0.00034	0.00037	0.00256
Horizontal	Deg.	0.00000	0.00034	0.00070	0.00997
Vertical	Deg.	0.00000	0.00024	0.00028	0.00196
Distance mm OR ppm	mm	0.00001	0.01605	0.00659	0.00418
	ppm	0.00019	2.84723	1.11790	0.65631

Table 3 presents summary statistics for the uncertainty magnitudes for the 1497 points as computed by SpatialAnalyzer. Please note the uncertainty analysis values are presented as 1-sigma confidence intervals [4].

Table 3: Coordinate Uncertainty, summary statistics comparing weighting schemes.

		SET 1	SET 2		
Run		1	1	2	3
Average	mm	0.0027	0.0171	0.0106	0.0048
Stdev	mm	0.0001	0.0043	0.0051	0.0002
Max	mm	0.0032	0.0377	0.0397	0.0056

Histograms from each set of coordinate uncertainties were created and shown in Figure 4. The magnitude of the uncertainty profile was used for this comparison. Figure 5 shows the cumulative percentage relative to the uncertainty.

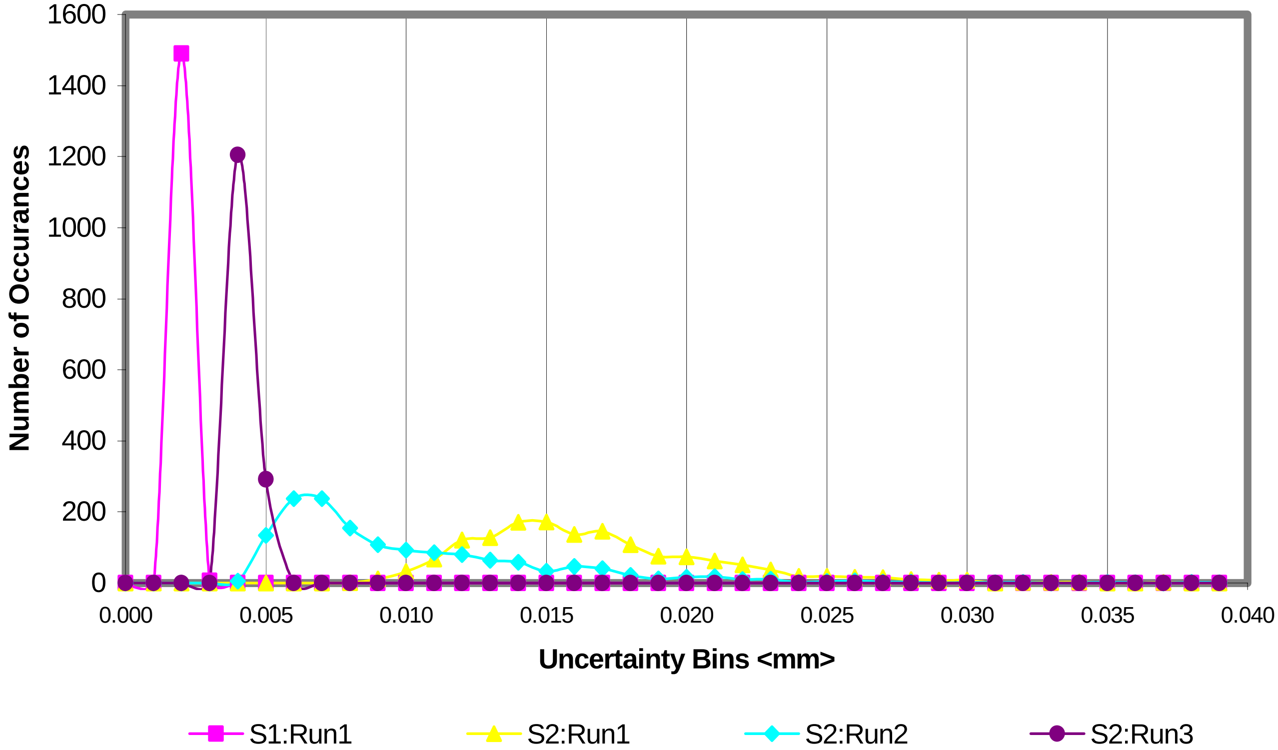

Figure 4: Coordinate Uncertainty Histograms, comparing weight schemes.

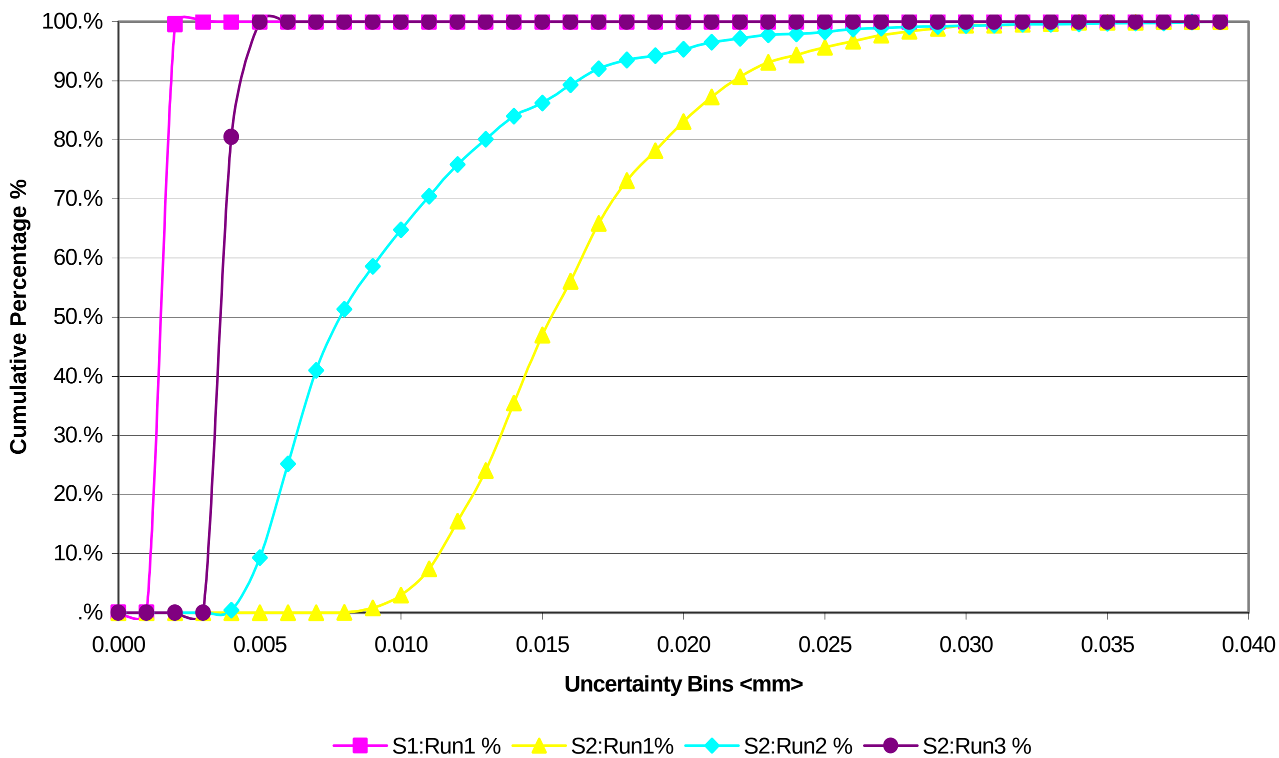

Figure 5 Cumulative Percentages for Coordinate Uncertainties, comparing weighting schemes.

The histogram and cumulative percentage charts show the difference between weighting schemes. It is apparent from the charts that reducing the dependence on the angle components can yield improved uncertainty. When the angles are effectively weighted out of the solution the average 1-sigma uncertainty value for data set 2 (i.e., 0.0048 mm), meets the prescribed uncertainty requirement of 5 microns [4].

The computed coordinates were then compared to the nominal point coordinates. This step in the analysis provides an objective means for evaluating the uncertainty values and the performance of the relative angle vs. range weightings. The results are shown in Table 4. Figure 5 and Figure 6 show a histogram and cumulative percentage charts for the noisy data. The prefect dataset is not shown because the coordinate residuals were effectively zero, i.e., the residuals were on the same order of magnitude as the number of significant digits in the nominal data.

Table 4: Nominal to Computed Coordinate Comparison, summary statistics comparing weighting schemes.

		SET 1	SET 2		
Run		1	1	2	3
Average	mm	0.0000	0.0213	0.0117	0.0291
Stdev	mm	0.0000	0.0233	0.0128	0.0325
Max	mm	0.0000	0.0557	0.0319	0.0873

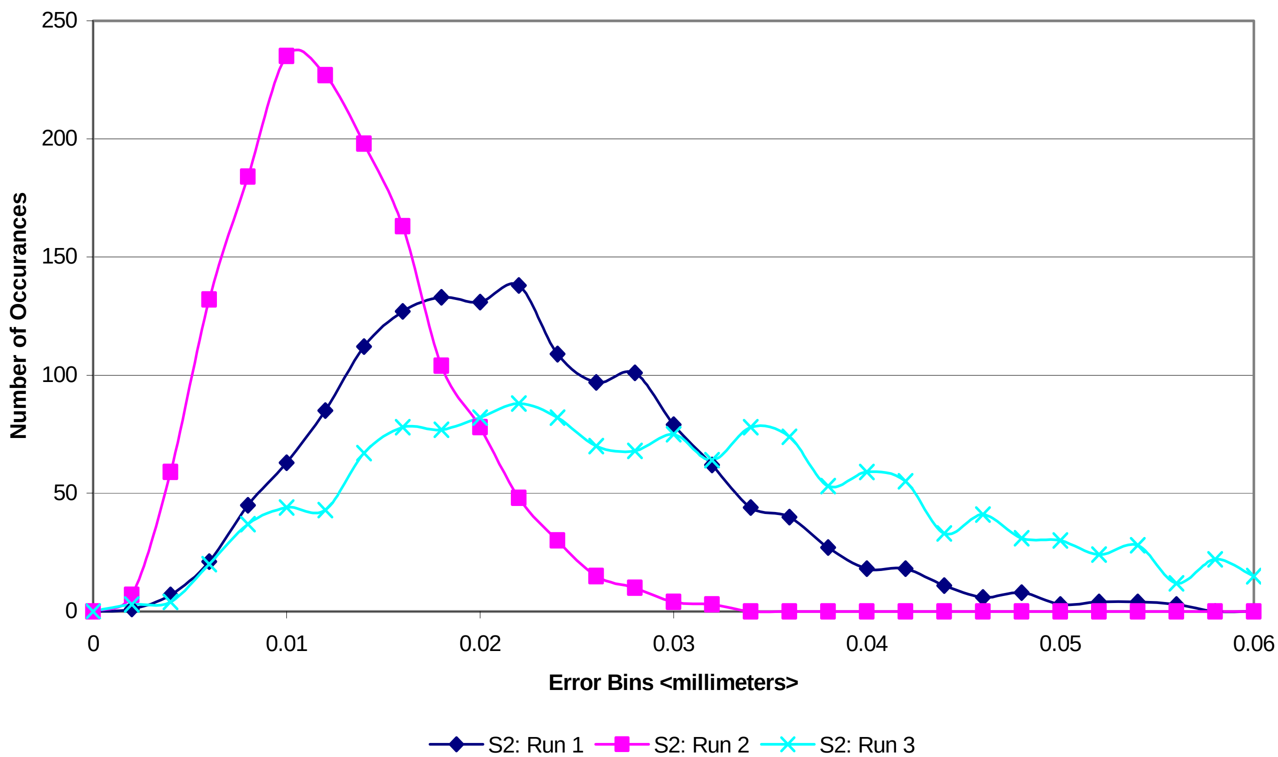

Figure 6: Nominal to Computed Coordinate Comparison Histograms, comparing weight schemes.

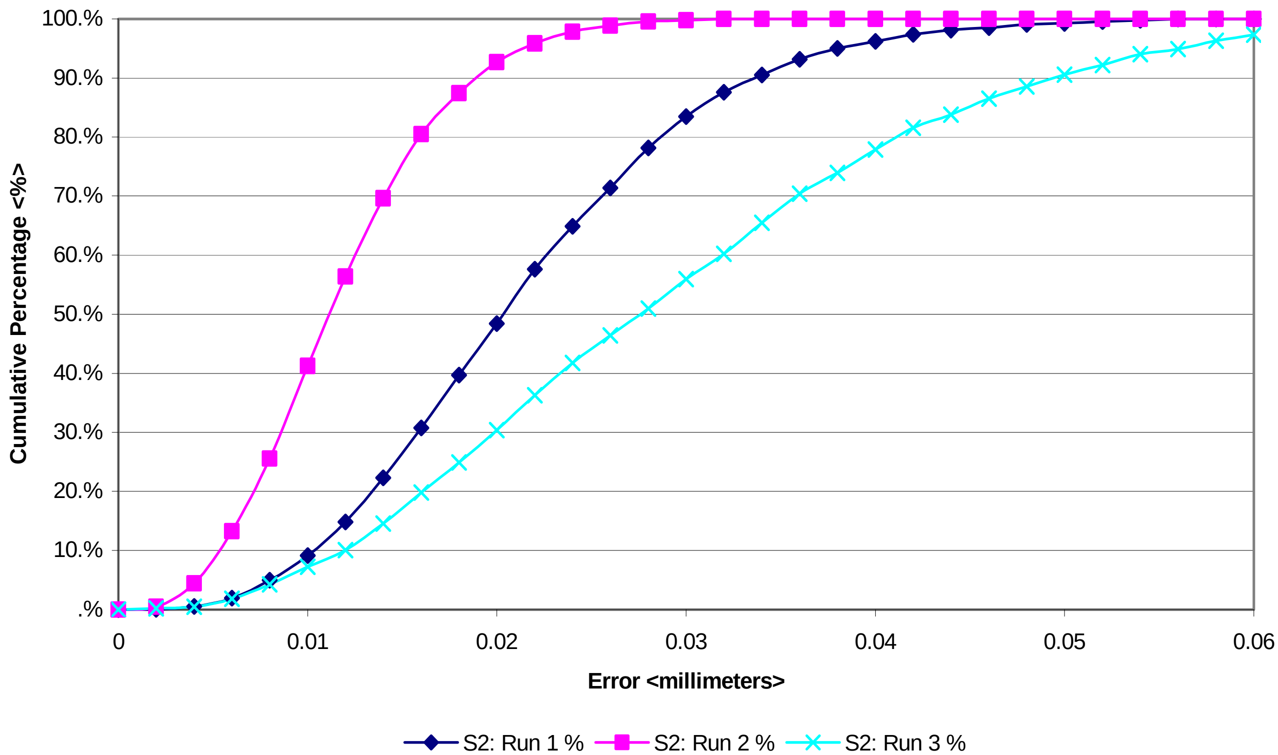

Figure 7 Cumulative Percentages for Nominal to Computed Coordinate Comparison, comparing weighting schemes.

Analysis of the comparison to nominal results shows a correlation between the different weighting scheme bundle residuals and the difference between the nominal points and their computed coordinates. The optimum results were achieved when the relative angle weight was 0.2 verses the range weight of 1.0, (i.e., Set 2, Run 2). The reported uncertainties results when the angle weights were 0.2 where within 0.004 millimeters of the comparison to nominal result. This is approximately 1 part per million relative to the range between the trackers and the simulated points. The uncertainty results for the other test cases, angles weighted at 1.0 and 0, did not match their comparison to nominal results. This result was due to the nature of the noisy added to the nominal data. The noise in this test case more closely modeled the Run2 relationship.

Test Procedure

Distances between Laser Trackers

Since the data for the simulation was developed in terms of a distance and two angles from a laser tracker to each data point, there was no inherent reference frame. An arbitrary coordinate system can be defined by using one of the laser trackers as the origin. The distances between pairs of laser trackers is a quick check of the quality of the solution since the distances between points is independent of the coordinate system. A comparison was done of the distances between the actual positions of the laser trackers and the distances between the derived positions. For the data set without noise, the derived distances are within 0.1 microns of the actual distances and for the data set with noise the derived distances between trackers was within 10 microns of the actual distances.

The transformation between the initial and derived coordinate system depends on three rotations and translations in x, y, and z. A 4 by 4 homogeneous transformation matrix was used (Foley and Van Dam, 1982) [8]. A major advantage of using homogeneous coordinates and transformations is that translations and rotations can be combined into one 4 by 4 transformation matrix.

A point at (x, y, z) is represented by a row vector of the form:

$$(x, y, z, 1).$$

The fourth element is one (1) because of the 4 by 4 homogeneous transformation matrices that are used.

The total transformation matrix, T, including rotations and three translations, Δx, Δy, and Δz has the form:

$$T = \begin{bmatrix} \cos(\varphi)\cos(\psi) - \sin(\phi)\cos(\theta)\sin(\psi) & -\cos(\phi)\sin(\psi) - \sin(\phi)\cos(\theta)\cos(\psi) & \sin(\phi)\sin(\theta) & 0 \\ \sin(\varphi)\cos(\psi) + \cos(\phi)\cos(\theta)\sin(\psi) & -\sin(\phi)\sin(\psi) + \cos(\phi)\cos(\theta)\cos(\psi) & -\cos(\phi)\sin(\theta) & 0 \\ \sin(\theta)\sin(\psi) & \sin(\theta)\cos(\psi) & \cos(\theta) & 0 \\ -\Delta x & -\Delta y & -\Delta z & 1 \end{bmatrix}$$

Where the 3 by 3 submatrix on the upper left is only a function of the 3 Eulerian angles, ϕ, θ,and ψ that are defined in Goldstein (1950) [6].

Solution for the Transformation Matrix

The actual solution for the transformation matrix between a set of points in two different coordinate systems $(x_1, y_1, z_1, 1)$ and $(x_2, y_2, z_2, 1)$ is straightforward.

Read in data set in coordinate system one:

$$D_1 = \begin{bmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ \dots & \dots & \dots & 1 \\ x_n & y_n & z_n & 1 \end{bmatrix}$$

The data format is a *n* by 4 matrix with each point corresponding to a row of the matrix. The jth row has the format:

$$(x_j, y_j, z_j, 1).$$

Read in data set in coordinate system two, D_2, which has the same format as D_1.

Set the design matrix (Press, et al., 1992) [7], *A*, for a linear least squares fit equal to:

$$A = D_1$$

Derive the covariance matrix, *cv*, from the design matrix:

$$cv = \left(A^T \cdot A\right)^{-1}$$

Then the required transformation matrix is:

$$T = cv \cdot A^T \cdot D_2$$

Which has the form:

$$T = \begin{bmatrix} t_{1,1} & t_{1,2} & t_{1,3} & 0 \\ t_{2,1} & t_{2,2} & t_{2,3} & 0 \\ t_{3,1} & t_{3,2} & t_{3,3} & 0 \\ -\Delta x & -\Delta y & -\Delta z & 1 \end{bmatrix}$$

Solve for the three rotation angles.

The upper left 9 elements, $t_{1,1}$ through $t_{3,3}$, of the transformation matrix, T, are not independent as they are functions of the 3 Eulerian angles: ϕ, θ,and ψ. Solve for these angles:

$$\phi = \text{atan}\,2(t_{1,3}, t_{2,3})$$

$$\varphi = \text{atan}\,2(t_{3,1}, t_{3,2})$$

$$\theta = \text{asin}(t_{3,1}, \sin(\varphi))$$

Where the atan2 parameters are atan2(y, x).

Use the above rotational angles, and translations as input to a non-linear optimization routine. Typically, the Eulerian angles are within 0.05 degrees, and the displacements are within a few microns of the optimum values.

Shop Floor Experiments

A number of experiments have been conducted on the factory floor with three and four simultaneously connected laser trackers. Figures 6 through 9, show two of the configurations. Leica instruments are shown in these images, SMX and API instruments could have been in the same network.

Figure 8: 4 Laser Tracker Test Setup

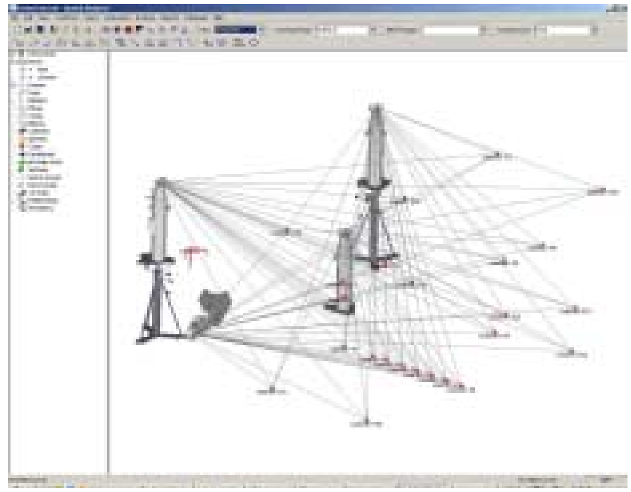

Figure 9: Spread configuration for Trilateration Test

Figure 10: Inline configuration for Trilateration Test

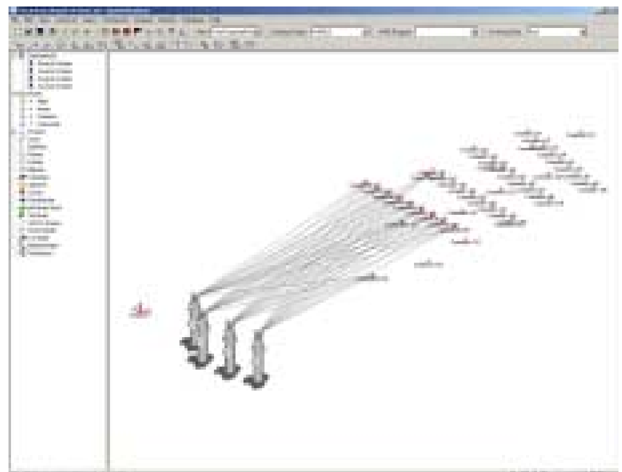

Figure 11: Screen shot of Inline test configuration.

A cats-eye prism was used as the reflector in each test case to expand the acceptance angle when trilaterating. The cats-eye prism has an acceptance angle of approximately ±60 degrees. Typical open-air corner-cubes have approximately ±40 degrees of acceptance angle [6]. While the acceptance angle is greater for the cat-eye prism, the reflector is bigger and heavier and therefore has a larger offset and is more likely to induce systematic reflector error into the measurements [6]. Care should be taken to ensure a consistent reflector orientation is maintained through the survey to minimize systematic error induced by the reflector. [5,6]

The when configured in a three-tracker trilateration network (as shown in Figure 12) the test results yielded an average uncertainty of 0.0075 millimeters (2-sigma) for the 30 targets and 90 measurements. The maximum uncertainty was 0.0085 millimeters (2-sigma). The angles were weighted at 0.01 verses the ranges at 1.0. The results of the bundle and the component uncertainties are shown below (see Table 5). The trackers were approximately 4 to 5 meters from the targets and 6 meters between themselves.

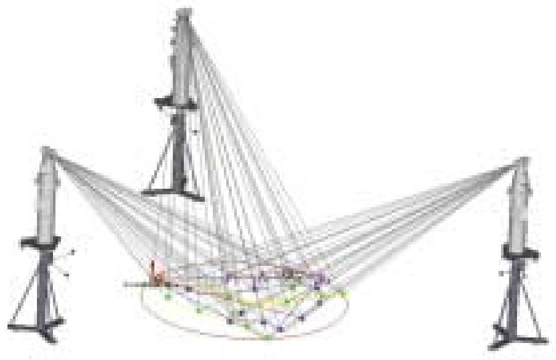

Figure 12: Three-tracker trilateration network

Table 5: Bundle results for the 3-Tracker Trilateration Network

Overall Uncertainty Analysis: (1 Sigma)
Angular: 90 measurements
Theta or Horizontal u = 0.000252 deg. (0.908214 arcseconds)
Phi or Vertical u = 0.000313 deg. (1.126698 arcseconds)
Distance: 90 measurements
u = 0.000005 (job units) OR u = 0.001120 ppm (millimeters)

Applications

An application for simultaneous real-time laser tracker metrology was completed in July of 2000. It utilized a weight bundle and simultaneous data collection from 2 trackers to test of the relative misalignment between the upper and lower machine heads of an ultra-sonic test machine head. The positions of the two machine heads that hold the transmitter and receiver nozzles were independently measured by two laser tracking interferometer systems in a common coordinate system as they moved at rates of 10 inches per second. (ips) A comparison of the relative head positions quantified the degree of dynamic misalignment between the machine heads.

Figure 13: Simultaneous tracking application to dynamically test machine head alignment.

Figure 14: Test setup for the dual tracker machine verification test.

Figure 15 is a screen shot from the SpatialAnalyzer software showing the two laser trackers and the two planes of dynamic measurements of the machine heads. The extra points, in the middle of the figure were process points (i.e., common points) that were used to align the laser trackers into a consistent coordinate system.

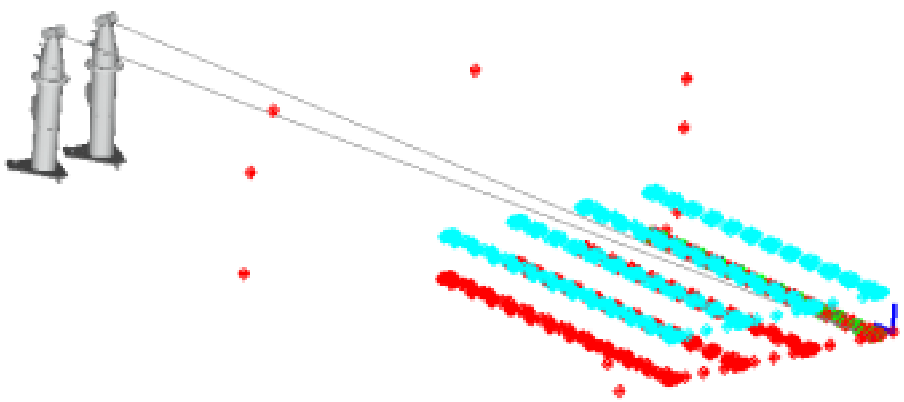

Figure 15: Test setup of the two laser trackers, measurement points, and machine bed coordinate system frame.

The evaluation of the TTU dynamic head misalignment found the error in the X-axis to be between +0.106 and -0.062 inches (positive indicates the lower head was out in a more positive x position than the upper head) when the system was moving at 10-ips. The data from the X-axis indicates the average error was 0.036 inches, suggesting the lower head generally leads, while the upper head generally lags by 0.036 inches. Figure 16 charts these results. The standard deviation of the X-axis error suggests the heads are misaligned within 0.099 and -0.027 inches in the X-direction about 95% of the time at 10-ips. The Y-axis errors are roughly two-thirds of the magnitude of the X-axis errors. The lower head generally lead the upper head by 0.019 inches in the Y-direction. The Z-axis errors were roughly one-third the X-axis errors.

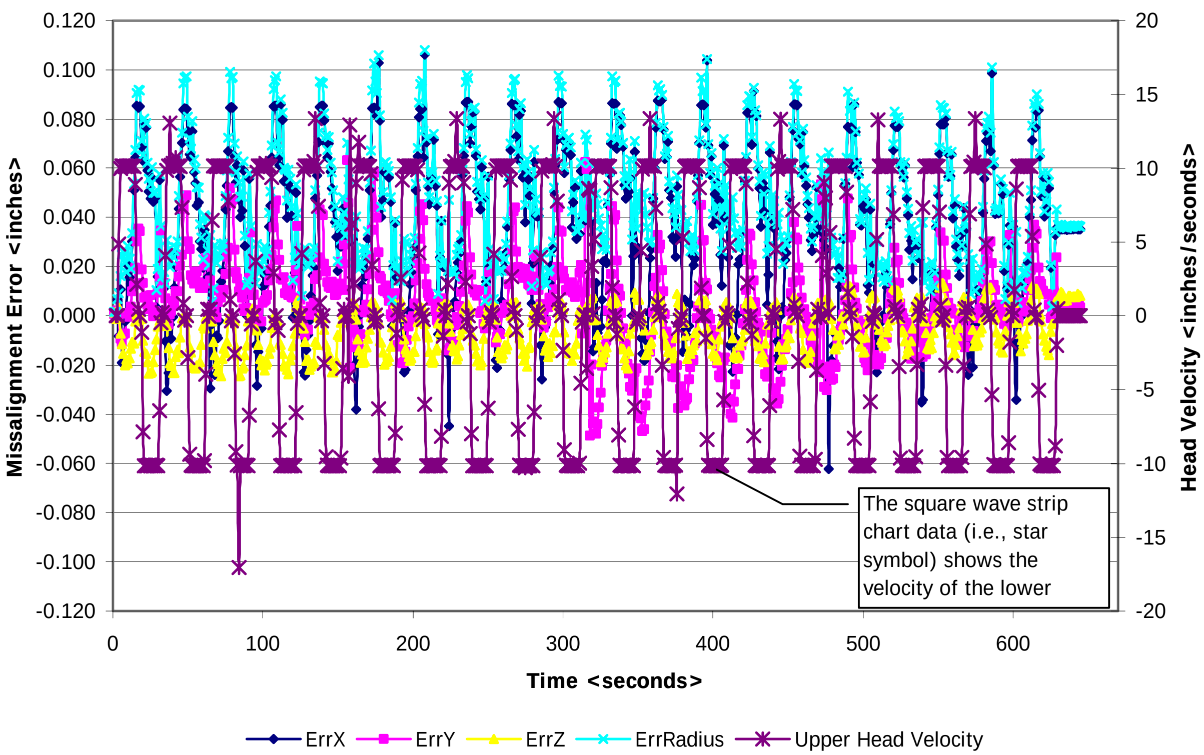

Figure 16: TTU Machine Misalignment

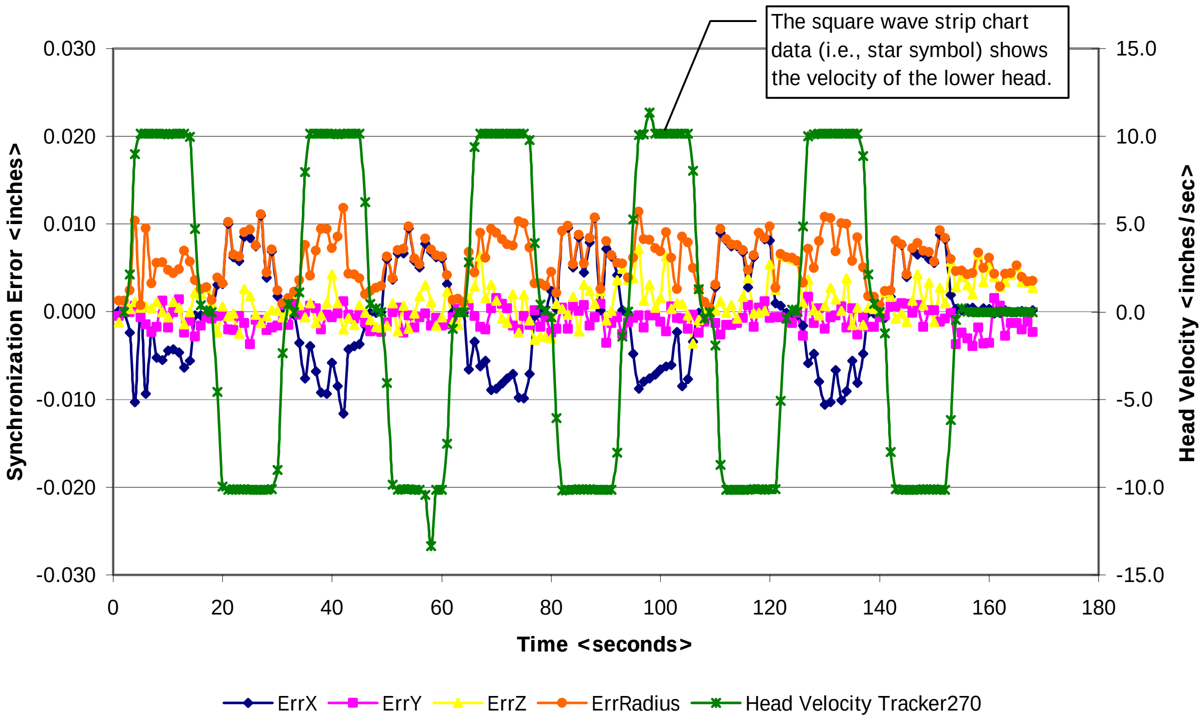

Figure 17: Synchronization Error between the Laser Tracking Interferometers

The measurements from the two laser-tracking interferometers were synchronized within two milliseconds by the SpatialAnalyzer software package, as seen from the data in Figure 17. A test where both trackers tracked the same retro-reflector on the machine was run to assess the measurement error for this measurement technique. The results indicate the one to two millisecond synch error combined with the dynamic measurement errors in the laser trackers show the measurement error was less than +/- 0.012 inches. The positions of the two machine head were recorded every second throughout the working envelope of the machine as it moved at rates of 10-ips. The machine took 11:36 (m:s) to move through its work envelope at 10-ips.

Conclusions

Weighted trilateration numerical methods can improve measurement uncertainties with commercial Laser Tracking Interferometer systems on large parts to 5 microns (1-sigma). Weighted trilateration calculations use distance and angle measurements to determine the three-dimensional coordinates of unknown positions. These calculations give higher weight to the interferometer distance measurement from laser trackers; verses the weight given to the laser tracker's angle measurements. This solution technique effectively provides a more accurate measurement for several reasons, it optimizes the instruments strengths, it uses the extra information (i.e., angles) to strengthen the solution, and it provides for robust error checking. Laser Tracker Trilateration positioning systems can be used in applications where the use of other positioning systems is impractical. Since the measurement instruments are portable and are able to track and report results in real-time, they can be used in virtually any location to provide very accurate large-scale measurements of dynamic events.

The weighted trilateration mathematical solution was discussed and shown to produce uncertainties at the 5-micron level in simulated and shop floor experiments. Shop floor tests were completed with three and four simultaneously connected laser trackers using the SpatialAnalyzer software platform. An application using the synchronous laser tracking interferometers to evaluate the dynamic behavior of a two-headed inspection system, indicate the systems had a one to two millisecond synch error.

Acknowledgements

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