A Practical Method for Evaluating Measurement System Uncertainty

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Abstract

This paper describes a procedure used to determine the measurement uncertainties of spherical measurement systems. This procedure requires measuring a series of unknown yet fixed points from several different instrument locations. The resulting observations are then bundled to determine the set of instrument locations that minimizes the observational discrepancies of the system. We shall consider a total measurement to be composed of three independent measurements; horizontal angle, vertical angle, and distance. The goal of this process is to derive experimentally, the individual uncertainties associated with each of these three components. By statistically processing the post-bundle discrepancies for each observation, estimates of the overall uncertainty for each component value can be obtained. We will then present how the individual component uncertainties can be used to produce realistic estimates of total coordinate uncertainty.

One primary benefit of this method is that it does not attempt in any way to isolate the instrument from the measurement environment. Instead it considers the entire metrology system, including the operator, atmosphere, and targeting (SMR's). Poor operator technique, temperature gradients and other adverse environmental effects, and poor targeting will effect the performance of the system. With this method, these effects will manifest themselves as an increase in the H, V, D uncertainty values. It is important to note that this approach will not identify the cause of poor performance, but will present an accurate and repeatable measure of how well (or poorly) the system is performing. Poor results, indicate poor system performance, not necessarily poor tracker performance. Good results, indicate good performance of the entire system.

1 Introduction:

Many different methods are used to present uncertainty specifications for measurement systems. Most often, manufacturers seek to provide laboratory results that isolate the instrument from "real-world" effects such as operator technique and environmental instabilities. This portrayal of the instrument is necessary to ensure proper comparisons of various instrumentation, but this portrayal does not indicate the level of performance an average operator can expect in a less than pristine environment. As a result, users often have difficulty evaluating or even estimating the performance of their instrument under realistic conditions. Most of the instrument specifications do not quantify uncertainty resulting from factors such as operator technique, environmental properties, and targeting issues. All of these issues must be considered in evaluating the total system performance. This work seeks to develop a method to characterize the working uncertainty of the device so users will have reasonable expectations of device performance in a variety of situations.

Many current evaluation methods make use of known artifacts. These artifacts are often expensive and difficult to maintain and transport. We chose instead to use a series of fixed, but unknown targets for this procedure. The performance of the instrument is evaluated based on the fact that the points remain fixed throughout the measurement process. The degree to which the instrument shows the points to move when they are measured from different instrument locations is directly related to the uncertainty of the system. If the system had zero uncertainty, the measurements of the points would be identical for all instrument locations.

Although, this method is applicable to many types of instrument systems, we have used it specifically to evaluate spherical devices including theodolites, distance measuring theodolites, laser trackers, and laser scanners. In this paper we will focus specifically on laser tracker systems.

2 Measurement and Analysis Procedure

There are three basic steps to this method. First, a series of fixed targets must be measured by the device from several different instrument locations. Second, the resulting horizontal, vertical, and distance observations are bundled to determine the location of each instrument that minimizes the overall system error. Third, the residual errors of the bundle adjustment are statistically processed to determine the uncertainty of the measurement system's output values. In this section we will also extend the instrument uncertainty analysis to actual coordinate uncertainty using simulation methods.

2.1 Acquiring the Measurement Data

The data acquisition procedure is relatively straightforward and may be accomplished with suitable care in as little as 30 minutes. Operators should resist the inclination to perform these measurements with greater care than would be used for an actual measurement process.

The first step is to establish a static point field for observation. Typically, a series of rigid points are used (SMR nests in the case of laser trackers) in an arrangement that exercises the device throughout the workspace of interest. It is recommended that at least ten fixed points be used. In theory, the presence of more points would yield better results. However, simulation results indicate drastically diminishing returns above ten targets. Simulations of 500 target fields and ten target fields yield practically identical results.

Measure each target from the current instrument location. Move the instrument to another location, and measure each target again. Repeat this process for 7 instrument locations. It is important to distribute the instrument locations throughout the workspace. We suggest raising and lowering the instrument, rotating it, and moving it to locations around and within the area of interest. The goal is to exercise the working volume as much as possible. In practice, we have found that after only 4 instrument plants, the bundle uncertainty values seem to converge fairly close to the final values after all of the locations have been completed. This means that for quick on-site verification checks, the full set of instrument positions may not be necessary.

At the end of the measurement process, you will have a set of H, V, and D values for each of the targets in the fixed field measured from each instrument location. It is important to get the actual HVD values after kinematic compensation, not the coordinate values since we are interested in bundle adjusting the results, not best-fitting the points together.

2.2 Performing the Bundle Adjustment

In order to evaluate the errors in the measurement system, we must determine the transformation of each instrument plant in the measurement process. To do this, we use a bundle adjustment algorithm. This consists of an optimization routine that determines the transformation of each instrument that will minimize the observational errors of the system.

2.2.1 Bundle Algorithm

The error function that is used for the bundle is shown in Figure 2.1. In this planar example, only 2 error components are shown though for the actual 3D case, there is also a vertical angular component, $\mathbf{eV_2}$.

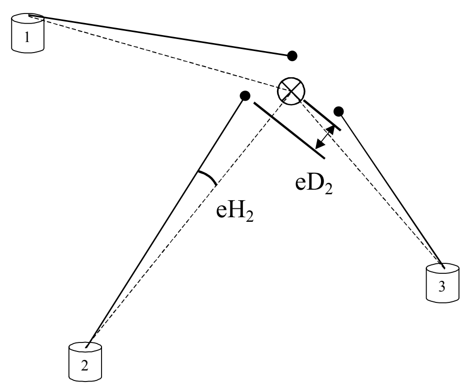

Figure 2.1: Bundle Residual Components

For this example, each of the three measurements of the point contribute 3 individual error terms, **eH**, **eV**, and **eD**. This means that there are 9 total error values added to the objective function for this point.

Since **eD** is expressed in units of length, and the other components are expressed in angular units, we convert the distance error to an angle using the arctangent of **eD** divided by **D**, the measurements' actual length value. This allows the objective function to remain homogenous in units.

Within the bundle adjustment optimization there is a nested point location optimization. This is necessary because for each set of measurements at a given point, we must determine the XYZ point location that best represents all the measurements. The point location optimization uses the same error function as the bundle, but instead of moving the instruments, it moves the point to minimize the error. Once the point is in an optimal location, the error values are used to determine how to move the instruments to further reduce the errors. When the instruments are moved, the point location optimization is repeated. This process continues until none of the attempted moves allow any improvement in the overall objective function.

This method had long been used for theodolite systems with the addition of a scale bar to provide scaling for the system. The difference is that only the H and V components are used in the bundle since there is no D measurement. Otherwise the process is identical.

It should be noted that an additional constraint set could be added by incorporating a "scale-bar" into the laser tracker bundle as well. A precision interferometer could be used to measure the distance between any two target locations and thus supply an independent check of scale, divorced from the instrument in question. By weighting this scale-bar error very heavily in the optimization, any distance anomalies of the instrument would become apparent. This is analogous to adding a known artifact to the process.

2.2.2 Optimization Weighting

As with any numerical optimization procedure, it is possible to affect the outcome by changing the weighting values. In the case of bundle adjustment, many different weighting schemes may be used. For the laser tracker bundles presented in this paper, we chose to weight the angle residuals by 0.5 and the distance residuals by 1.0. The reasoning behind this is that generally speaking, the distance measurement capabilities of the laser tracker exceed its ability to resolve angles. Recall that the distance errors are expressed in angular units so the weights are not needed to make the units comparable, but instead to set the relative importance of the error components.

The case where the angle weight is zero (or nearly zero) and the distance weight is 1.0 is particularly interesting since it determines the point locations using multilateration. Since the distance measurement capabilities of the laser tracker are known to exceed the angular capabilities, multilateration should yield the most accurate coordinates for the points.

2.3 Computing Uncertainty from the Bundle Residuals

When the bundle adjustment is completed, each point will have multiple rays defining measurements from the various instrument locations. The amount these rays do not meet the optimal target location defines the error in the system. This error is computed in each iteration of the bundle adjustment.

The next step in the uncertainty characterization process is to group all the H errors together, all the V errors together, and all the D errors together then process them statistically. The method we chose was to compute the standard deviation of the errors, assuming they are already mean-zero. The standard deviation value is then used to represent the one sigma uncertainty for the corresponding component. Other methods for analyzing the residual errors could also be employed. The key concept behind this method is not the specific statistical processing, but the use of the residual errors as a measure of uncertainty.

3 Sample Data

This section will present sample bundle adjustment data and the corresponding analysis. This particular bundle was performed using eight plants of a laser tracker each measuring sixteen fixed points. A weighting scheme of 0.5 for the horizontal and vertical components and 1.0 for the distance component was used for this example. The bundle adjustment and coordinate uncertainty analysis was performed using the SpatialAnalyzer metrology software package. The uncertainty test geometry is shown in Figure 3.1.

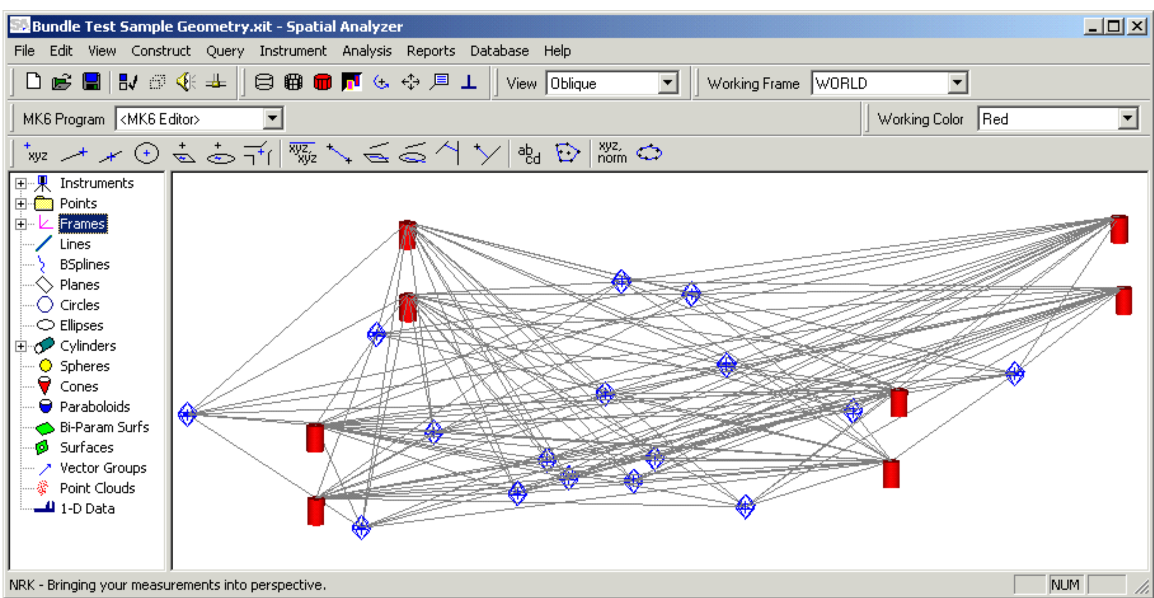

Figure 3.1: Sample Bundle Test Geometry

3.1 Bundle Adjustment

Before performing the bundle, it is necessary to have an initial guess that places the instruments in approximately the correct location. Without this step, the bundle may tend to get stuck in local minima. For this case, we chose to automatically determine the initial guess by best-fitting all of the data sets to the data set from the first location. From that location, a bundle optimization was initiated.

After determining the optimal instrument locations, the residual errors were grouped by component (H, V, or D). The histograms in Figure 3.2 show the distributions of the errors.

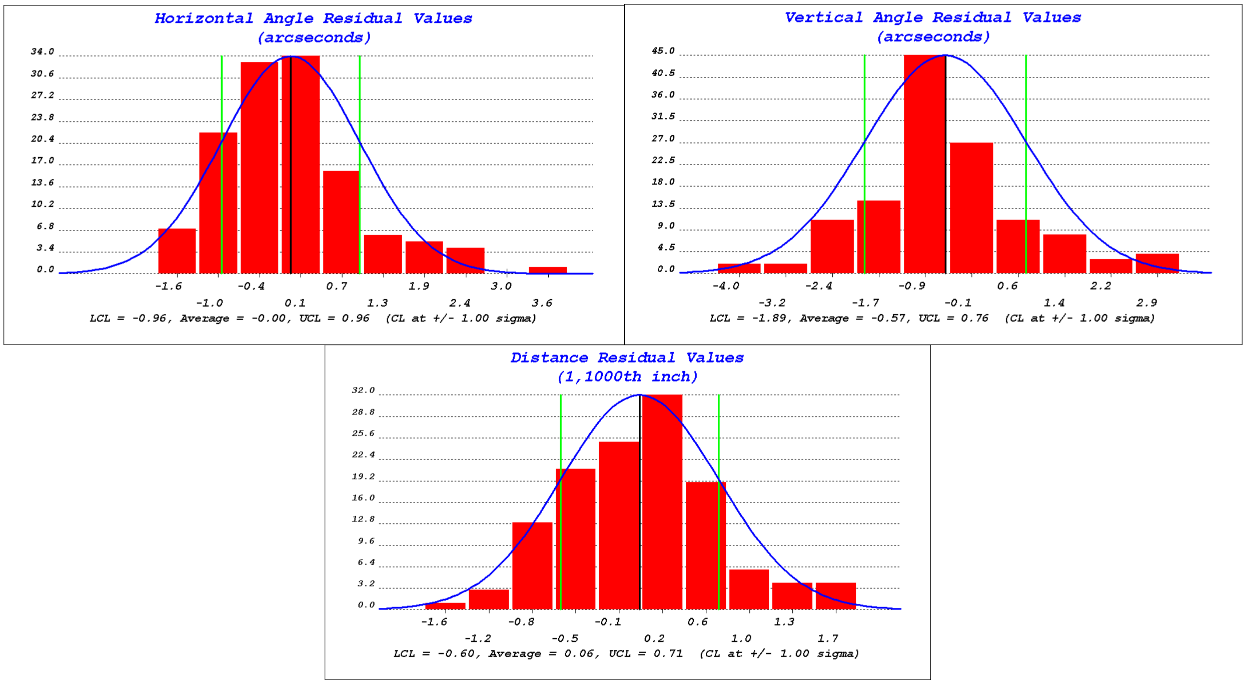

Figure 3.2: Bundle Residual Values

In this case, the statistical analysis of the residuals yielded the following (one sigma) results:

Horizontal Angle: 0.96 arcseconds
Vertical Angle: 1.44 arcseconds
Distance: 0.000658 inches.

Each value computed above was based on the standard deviation of 128 residual error values assuming that the residuals were already mean-zero based. The number of values results from eight instrument locations and sixteen points making the number of observations is 8 x 16 or 128.

To demonstrate the effect of the component weights on the uncertainty results, this data was also bundled using a variety of weighting values. Table 3.3 shows the results for other weights.

	Weights:			
	A=0.5, D=1	A=1, D=0.5	A=1, D=1	A=0, D=1
Horizontal (arcsec.)	0.96	0.76	0.82	1.57
Vertical (arcsec.)	1.44	0.29	1.34	3.25
Distance (1,000th in.)	0.658	1.346	1.116	0.191

Table 3.3: Bundle Results for Various Weighting Combinations

3.2 Coordinate Uncertainty Using Bundle Results

Often, a user is most interested in the XYZ uncertainty of the coordinates they are measuring or are planning to measure. Since the bundle algorithm determines the uncertainty in terms of the instruments HVD measurements, it is necessary to map these values to coordinate uncertainty. By using the uncertainty results from the bundle in an error simulation algorithm, it is possible to calculate the coordinate uncertainty using the HVD instrument uncertainty.

To do this we simulate the uncertainty of the instrument's values using a random distribution within the uncertainty bounds determined by the bundle. In this case, the horizontal uncertainty was computed at 0.96 arcseconds. This means that we apply a random distribution of errors to the horizontal measurement ranging from –0.96 to +0.96. We determine this distribution using both a Gaussian model as well as a rectangular model.

A simulation is performed using a large number of samples for each point. For each sample, random error is injected into all components of the measurements using the distributions discussed above. The result is a cloud of points surrounding the actual point. The clouds for this example are shown in Figure 3.4 with enhanced scale for visibility.

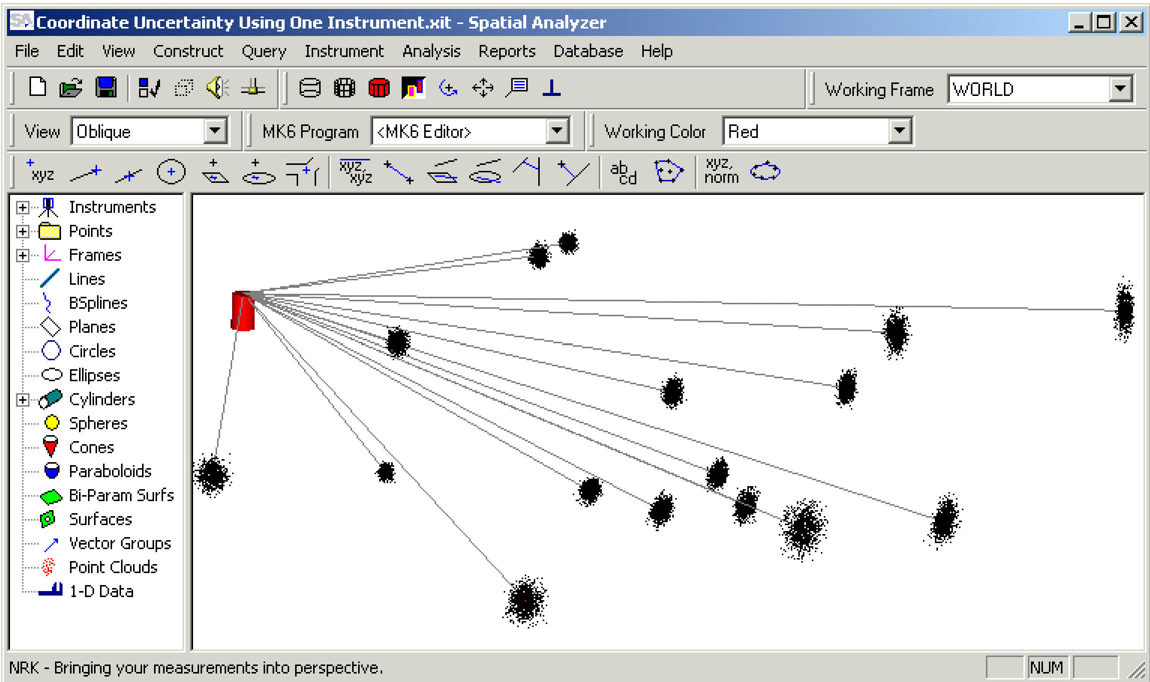

Figure 3.4: Uncertainty Clouds (Gaussian) for Bundle Test Points

Once the uncertainty clouds are constructed, we process them statistically to determine the coordinate uncertainty in terms of XYZ. To do this, we compute the standard deviation of all the X values, all the Y values, and all the Z values. The result is an uncertainty in each coordinate direction. The magnitude of this uncertainty vector may be used to represent the total uncertainty at the point.

Table 3.5 lists the uncertainties for the individual bundle points. Only the magnitude of the XYZ uncertainty is listed, but it is shown based on both Gaussian and Rectangular simulated error distributions applied to the HVD values.

Uncertainty values based on a 68.26% Confidence Interval (1.0 sigma) 5000 simulated samples with Gaussian and Rectangular distribution All values in inches.			
	Distance from Tracker	Uncertainty Gaussian	Uncertainty Rectangular
bundle::1	277	0.0008	0.0014
bundle::2	125	0.0004	0.0007
bundle::3	172	0.0005	0.0009
bundle::4	138	0.0004	0.0008
bundle::5	199	0.0006	0.0010
bundle::6	350	0.0010	0.0017
bundle::7	200	0.0006	0.0010
bundle::8	245	0.0007	0.0012
bundle::9	265	0.0008	0.0013
bundle::10	227	0.0007	0.0012
bundle::11	219	0.0006	0.0011
bundle::12	265	0.0008	0.0013
bundle::13	370	0.0010	0.0018
bundle::14	319	0.0009	0.0016
bundle::15	349	0.0010	0.0017
bundle::16	435	0.0012	0.0021
Maximum	**435**	**0.0012**	**0.0021**
Average	**260**	**0.0008**	**0.0013**
Minimum	**125**	**0.0004**	**0.0007**
Span	**310**	**0.0008**	**0.0014**

Table 3.5: Coordinate Uncertainty Results

3.2.1 Interpretation of Bundle HVD Uncertainty Values

Since the bundle approach is based on optimization methods, it will attempt to minimize errors by manipulating the system. The result is a tendency to report uncertainty results that are actually better (lower uncertainty) than a user would see in practice.

This method distills all the error in the system into the output values of the measurement device. Since a laser tracker system provides the user with HVD values, this method computes a HVD uncertainty. This means than outside factors such as weather conditions, target motion, operator error, and other degrading effects will be manifest as increased uncertainty in one or more of the system components.

Some would argue that this is not "fair" to the instrument since a good instrument that measures extremely accurately in a lab may have less than ideal bundle results on the factory floor. The goal of this approach is not to evaluate the best possible performance of a given instrument, but instead to give the user a realistic picture of how well the instrument is measuring in its working environment. In many cases the results we have seen illuminate the importance for good operator practices. For example, consistently better results are achievable if the operator is extremely careful regarding the orientation of the SMR before measurement. Although this practice is routine in laboratory environments, it is seldom witnessed during field measurements. It is our opinion that the operator, the SMR, the atmospheric conditions, and the instrument, all constitute the measurement system. In practice, the laboratory specifications of performance of an *instrument* may have very little to do with the actual field performance of the *system.*

4 Results

In this section, we will present the results from the bundle data we have analyzed to date. It is important to note that these measurements were taken in different regions of the country, using different instruments, different point locations, different instrument locations, various environmental conditions, and various states of instrument calibration/compensation. To perform a meaningful comparison between individual trackers, it would be necessary to test them in the same conditions with the same instrument and point geometry.

For all of these data sets, we chose to run the bundle with an angular residual weight of 0.5 and a distance weight of 1.0. In addition, we state all results at the one sigma level. Figure 4.1 shows the relative uncertainty values for each manufacturer's device. Table 4.2 shows the individual bundle results that were used to form these averages. The manufacturers are identified as A, B, and C for the purpose of anonymity.

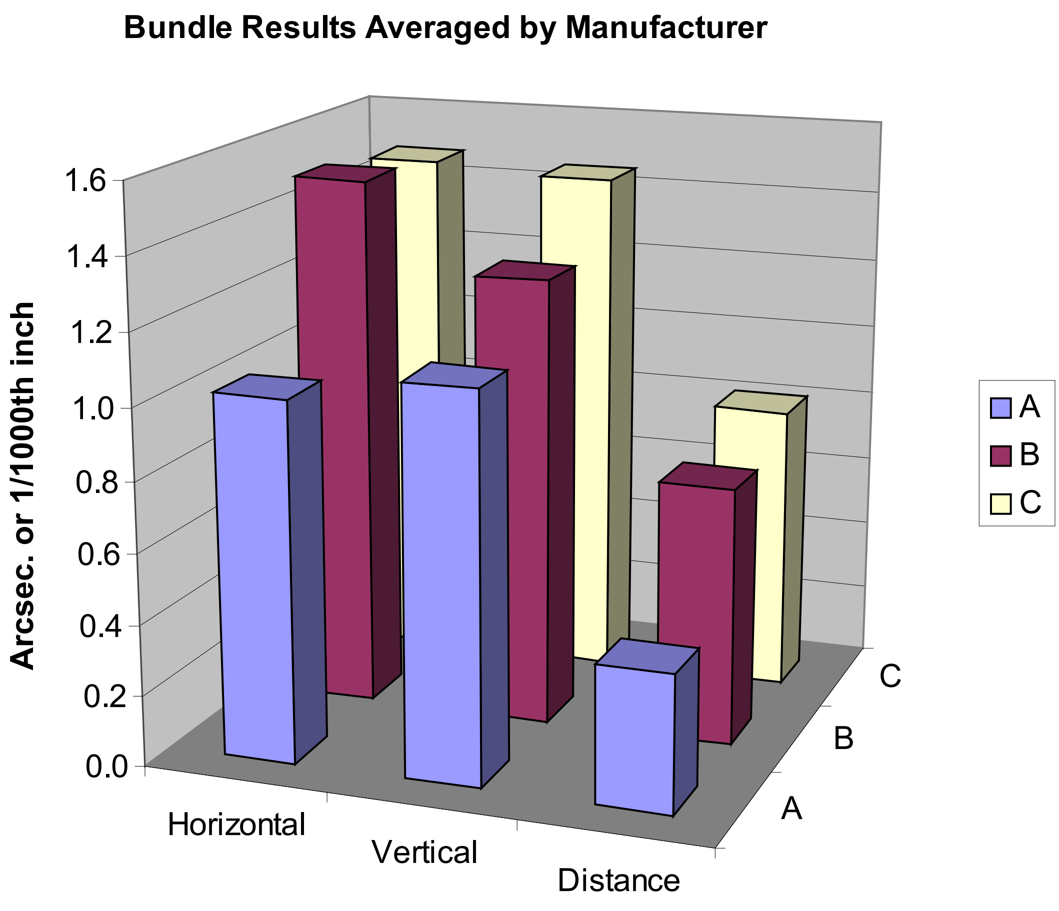

Figure 4.1: Bundle Results by Manufacturer

Manufacturer	Location	Horizontal (arcseconds)	Vertical (arcseconds)	Distance (1,000th inch)
A	1	0.6	0.4	0.2
	1	0.6	0.4	0.2
	2	1.1	1.5	0.5
	3	1.4	1.5	0.7
	4	1.3	1.7	0.3
	Average	**1.0**	**1.1**	**0.4**
B	1	1.8	2.9	1.5
	1	0.9	0.8	0.1
	2	1.0	1.4	0.7
	2	1.1	0.5	0.4
	3	2.2	1.3	0.5
	3	1.8	1.0	1.0
	3	1.8	1.0	1.0
	Average	**1.5**	**1.3**	**0.7**
C	1	1.8	1.9	1.0
	1	1.9	1.8	0.4
	2	1.0	0.9	0.4
	3	1.4	1.3	1.3
	4	1.2	1.3	0.7
	5	1.3	1.4	0.6
	6	1.7	1.5	1.4
	Average	**1.5**	**1.5**	**0.8**

Table 4.2: Individual Bundle Results

5 Conclusions

This paper presents an experimental method for determining the individual uncertainties associated with the H,V, and D values of a spherical measurement system. The method requires measurement of a fixed target field from several instrument locations. The measurement data is then bundled to determine the instrument locations that minimize the error in the system. By statistically processing the post-bundle discrepancies for each observation, estimates of the overall uncertainty for each component value can be obtained. Realistic estimates of X,Y,Z, and total coordinate uncertainty can then be derived using a simulation algorithm.

One primary benefit of this method is that it does not attempt in any way to isolate the instrument from the measurement environment. Instead it considers the entire metrology system, including the operator, atmosphere, and targeting (SMR's). Poor operator technique, temperature gradients and other adverse environmental effects, and poor targeting will effect the performance of the system. With this method, these effects will manifest themselves as an increase in the H, V, D uncertainty values. It is important to note that this approach will not identify the cause of poor performance, but will present an accurate and repeatable measure of how well (or poorly) the system is performing. Poor results, indicate poor system performance, not necessarily poor tracker performance. Good results, indicate good performance of the entire system.

This method has many applications. It can be used to quantify the degradation of measurement data due to environmental effects, operator error, and other factors. It may be used as a test of measurement system health on a periodic basis. Different measurement systems can be compared under identical conditions to determine their suitability for a particular measurement job. In this paper, we have presented a cross-section of results under varying conditions in order to provide an insight into the performance of laser-tracker systems in general.